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AGARD REPORT No.775

Rotorcraft Drivetrain Life Safety and Reliability

(Cycle de Vie, Sécurité et Fiabilité des Chaînes Dynamiques des Avions à Voilure Tournante)

NORTH ATLANTIC TREATY ORGANIZATION

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ADVISORY GROUP FOR AEROSPACE RESEARCH AND DEVELOPMENT

(ORGANISATION DU TRAITE DE L'ATLANTIQUE NORD)

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Rotorcraft Drivetrain Life Safety and Reliability

(Cycle de Vie, Sécurité et Fiabilité des Chaînes Dynamiques des Avions à Voilure Tournante)

by

D.Astridge and M.Savage

This Report was sponsored by the Propulsion and Energetics Panel of AGARD.

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AGARD R775 (June 1990)

Abstract

This report consists of two chapters of an originally intended comprehensive volume on Transmission Systems for Power Transfer in Helicopters and Turboprops which could not be prepared as a whole. After a short introduction, the first chapter deals with lessons learned from accident data of large civil transport helicopters, and with *recent improvements in design* and component technology, including recommendations and safety benefits achievable by monitoring systems. In the second chapter, statistics of drive system life and reliability were discussed, allowing a proper evaluation of drive system designs and the understanding and control of mean life and life scatter of a drive system at the design stage. [illegible]

This AGARD Report was prepared at the request of the Propulsion and Energetics Panel of AGARD.

[illegible] Transmissions; Helicopter rotor drives, [illegible] Helicopter engines; Turboshaft/turboprop engines; [illegible]

Résumé

Ce rapport consiste en deux chapitres qui devaient faire partie d'un ouvrage complet sur les chaines cinématiques de transfert de mouvement pour hélicoptères et turbopropulseurs, lequel n'a pu être réalisé.

Suite à une courte introduction, le premier chapitre traite des leçons tirées des expertises d'accidents concernant les hélicoptères lourds de transport civils, ainsi que les améliorations qui ont été apportées récemment dans le domaine de la conception et de la technologie des composants, y compris des recommandations concernant les systèmes de surveillance automatique et les gains qui en résultent sur le plan de la sécurité.

Le deuxième chapitre couvre les aspects cycle de vie et fiabilité des systèmes d'entrainement. Il permet de faire une évaluation des differents systèmes et de mieux comprendre et donc, de mieux contrôler, la durée de vie moyenne et la dispersion des prévisions de durée de vie d'un système d'entrainement pendant la phase d'étude.

Ce rapport AGARD a été réalisé à la demande du Panel AGARD de Propulsion et d'Energétique.

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INTRODUCTION

by

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Rotorcraft propulsion involves many complexities not associated with conventional lift flight. In addition to the technical challenges posed by the need to develop small, efficient turboshaft engines, efficient and flight worthy methods of transferring the mechanical power from one or more of these engines to the rotor system must be provided. Typically, the mechanical drivetrain of current production rotorcraft accounts for up to 20% of the airframe weight — a substantially larger share than accounted for by the engines. Of course, every effort is made during design to minimize drivetrain weight without compromising safety and reliability — drivetrain failure generally results in loss of aircraft. In fact, transmission and drivetrain problems are a significant cause of rotorcraft accidents which result in very substantial costs in dollars, human injury, and death. Transmission problems are also a significant cause for aircraft unavailability due to fleet groundings, mission aborts, and unscheduled maintenance operations which require time and money to take care of the necessary inspection, replacement, or repair.

In order to design safe, reliable, lightweight rotorcraft drivetrains, sophisticated and well-founded design procedures must be employed, and maximum benefit from advanced component, material, and lubrication technologies must be exploited. Together, the two articles comprising this report very effectively identify and discuss the important drivetrain life related technologies, and propose a thoroughly developed statistical life and maintenance prediction methodology upon which future drivetrain designs might be based. In addition, a concise treatment of rotorcraft safety, and the contribution of drivetrain failures to that safety record is provided.

The objective of this report is to provide a clear picture of how drivetrain technology fits into the overall rotorcraft reliability issue, and to identify key specific technologies and design methods that may potentially improve reliability. For the most part, the technologies identified are application ready and in many cases have already been applied in a piecemeal manner. Likewise, the basic principles of the statistical lifing approach outlined are not in themselves new, but do have yet to be applied in a systematic manner to a new transmission design.

Beyond the scope of this report are the advanced transmission configurations that hold considerable promise for reduced drivetrain weight and improved reliability. Those future gains would be achieved through not yet proven gear arrangements, application of not quite fully developed materials and components, and the use of clever design arrangements that eliminate many components and interfaces, but require some structural integrity validation.

It is anticipated that next generation rotorcraft drivetrains, employing the technologies and methods described in this report, in addition to some of the more advanced features, will comprise only 10—15% of airframe weight and demonstrate two to three times the reliability of currently produced drivetrains. Such improvements would have a major impact on aircraft efficiency and productivity, and would substantially reduce the life cycle cost of rotorcraft.

CHAPTER 1

HELICOPTER TRANSMISSIONS - DESIGN FOR SAFETY AND RELIABILITY *

D.G. Astridge, Derek Astridge & Associates, Langport, Somerset, England

SUMMARY

Recent improvements in design and component technologies are reviewed against a background of accident data analysis, resulting in grounds for confidence in higher safety levels in future rotorcraft transmission designs. Recommendations are made concerning the realisation of significant safety and reliability benefits afforded by effective health and usage monitoring systems.

The results of this study are applicable to all new aerospace gearbox applications including helicopters, tilt-rotor aircraft, advanced propeller engines, and accessory drive systems.

1. INTRODUCTION

The large disparity between the safety records of rotorcraft and fixed wing aircraft has been of major concern to several airworthiness authorities including the UK Civil Aviation Authority (CAA) (refs 1-3). This is recognised as being largely due to the uniquely hazardous tasks that helicopters can perform, the expansion of their use in transporting personnel to off-shore platforms, and to their greater mechanical complexity. In order to improve the safety of civil transport helicopters, the CAA are implementing medium and longer term changes to airworthiness requirements, including demands for much higher reliability levels in transmissions and rotor systems (3-fold improvement in the medium term, and a further 25-fold improvement in the longer term - ref.2). Helicopter transmissions have been identified as a major source of airworthiness related accidents and of unreliability. Engine reduction gears and accessory drive gears have also contributed to these problems, and many of the considerations in this paper will apply to these also.

The integrity, or the structural and functional soundness of helicopter transmissions is controlled by airworthiness authorities, civil, or military depending on the application. The controls applied normally concern:-

(a) Substantiation of design and manufacture.
(b) Maintenance procedures and schedules.

Both are applied to the Design Authority or constructor, generally by means of approval of the design and manufacturing organisations, by approval of required substantiation tests, and of maintenance provisions and manuals, including the identification of finite-life parts and of time between overhauls. Controls (b) are also applied to the operator.

With this in mind it is considered useful to examine possible design shortcomings in the past, as manifest in accidents and overhaul inspections and to examine the possible impact of recent improvements in design, component, and monitoring system technologies.

The scope of helicopter transmissions considered in this paper is that defined by the CAA (reproduced in Appendix 1)

It is stressed that the CAA, except where referenced, are not responsible for the accuracy of the analysis, lessons, nor conclusions presented by the author.

2. CONTRIBUTION OF TRANSMISSIONS TO HELICOPTER ACCIDENTS

A study of helicopter fatal accidents published in 1980 (ref.3) showed that transmission systems contributed to 22 per cent of airworthiness related accidents. That study covered primarily UK multi-engined public transport helicopters from 1964-1979 and was restricted to fatal accidents. In that period rotor systems were seen to be the prime cause of airworthiness related accidents (67 per cent). A survey of serious rotorcraft accidents involving fatigue fracture for the years 1937-1981, for

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all types, civil and military (ref.4) showed that engines and transmissions together accounted for 32 per cent of fatigue fracture accidents. Fatigue fractures should not automatically be attributed to design - they have sometimes resulted from inadequate maintenance or from overload of the helicopter. Data published in the CAA World Helicopter Accidents Summary (ref.5) relates to civil transport helicopters greater than 4,550Kg gross weight. Helicopter manufacturers and operators, where known, are identified in the CAA Summary, but in this paper are suppressed in the interests of encouraging wider debate of improvements in design, manufacture, and maintenance, particularly relative to health monitoring techniques. The Summary provides an analysis by helicopter type, but no analysis relative to component or cause. It does however, provide narrative for each entry, many of which are quite specific about the cause. Analysis of the usable narrative reports is summarized in Appendix 3 for the period 1956-1986, and the definition of an aircraft accident used by the CAA is reproduced in Appendix 2.

The principal component sources of potentially airworthiness-related accidents (design, manufacture, maintenance and undefined cause) for large/medium civil transport helicopters, world-wide, for the period 1956-1986 are seen from Appendix 3, (table 1) to be:-

*	Engines	28%
*	Rotors	27%
*	Transmissions	22%

Of the 21 accidents definitely attributed to transmissions, 13 related to design or manufacture and 8 to maintenance (table 2). The 13 accidents attributed to transmission design or manufacturing errors occurred in the period 1977-1986, the worst years being 1978 and 1983 (table 3). Whilst it may be of interest to examine manufacturer influence it is difficult to establish a reasonable basis on which to do this in the face of such a large number of factors of influence, including design date, number of aircraft in service, miles, usage rate, operating conditions, and potential operating abuse, but for what it is worth a significant variation is seen when the transmission design/manufacture related accidents are expressed in absolute terms or as a proportion of rotorcraft sold (table 4).

A large proportion of the total helicopter accidents reported in the CAA Summary did not have a definite cause or primary component determined, and the real total for transmissions and other components is therefore potentially greater than indicated. With the need to reduce airworthiness related accidents substantially (ref.2) the significance of transmission design, manufacture, or maintenance causes is sufficient to justify attention in all three areas.

3. LESSONS FROM TRANSMISSIONS-RELATED ACCIDENTS

It is stressed that this survey represents a general review of accident lessons 'from the side-lines', and that the primary mechanisms for implementing lessons from accidents are (i) by means of Airworthiness Notices, Bulletins, etc sent by airworthiness authorities directly to constructors and/or operators, and (ii) by constructors or operators taking voluntary actions based on accident investigations and their own data.

Analysis of the CAA Summary data (Appendix 3 - Table 5) reveals that the main problem area for transmissions on large/medium civil transport helicopters is external drive shafting and associated bearings - gearbox connecting shafts, rotor drive shafts, and gearbox input shafts. Of the internal components, gears head the list, followed by lubrication system components, bearings, and freewheels. In table 6 all of the transmission related accidents from reference 5, are summarised under component and cause headings and suggested lessons for design, manufacture, and maintenance, are indicated together with monitoring recommendations. Table 7 lists transmissions details from reference 4, which also covers smaller helicopters and military applications, but is restricted to fatigue failures, and contains insufficient information to permit detailed lessons other than those in 3.1 (vii) below.

3.1 Summarising the design lessons from accidents:-

(i) Overload:
Continuous in-service monitoring and analysis of torque-time exposure and cycles (usage monitoring) as a basis for the retirement of components is required to circumvent the frequent disparity between design assumptions and operational conditions.

(ii) Deflection:
Relative deflections and misalignments of external drive shafts, supporting structure, and panels in close proximity should be determined and demonstrated to be acceptable at all flight conditions.

(iii) Auxiliary Rotor:
Where practicable replace the tail or auxiliary rotor with alternative means of thrust reaction having inherently higher integrity and reliability. The McDonnell Douglas 'NOTAR' system (ref.6) may prove to meet such a requirement for smaller helicopters at least.

(iv) Lubrication - Drive Shafts:
Provide oil lubrication to splined couplings and shaft support bearings in preference to grease.

(v) Lubrication - Gearboxes:
Provide oil recirculation with redundancy/emergency features. Minimise pipework external to the gearbox, or ensure some component dipping at all times. Ensure effective oil contents gauging, and adequate oil filtration and wear debris capture provisions.

(vi) Component Wear/Fracture:
Provide effective health monitoring techniques capable of detecting all potential failure modes. Use better materials, finishing processes, lubricants, and cleanliness standards.

(vii) Stress Raisers:
Useful design pointers from reference 4 are the five most common initiation sites for fatigue cracks that have resulted in rotary wing accidents, not just confined to transmissions:
(a) Bolt, stud or screw
(b) Fillet, radius, or other stress concentration
(c) Corrosion
(d) Fastener hole or other hole
(e) Fretting

(viii) Spiral Bevel Gears:
Tooth design should be such as to ensure gear separating tendencies rather than pulling into mesh (ref.3).

(ix) Shim Material:
Solid ground spacers or shims are preferred to soft aluminium laminated shims (ref.3).

(x) Planet Gear Support:
Planetary gears running directly on rolling element bearings have exhibited fracture originating from stress concentrations at the ends of needle rollers, and from rolling contact fatigue with cylindrical and spherical rollers. This arrangement is common practice but is questioned by the author of reference 3. Effective means of detecting rolling contact fatigue is clearly essential for such arrangements, and stress concentrations must be avoided.

(xi) Design Modifications/Configuration Control:
Ensure that modifications introduced to overcome integrity problems on one application are universally applied (e.g. civil and military).

3.2 Summarising the manufacturing lessons from accidents:-

(i) Quality Control:
Several accidents were attributed to inadequate quality control in the manufacture of critical components and contacting parts.

(ii) Build Cleanliness:
Several wear/debris arisings - ensure highest standards of build cleanliness to reduce wear initiation, and blockage of oil jets and screens.

3.3 Summarising the maintenance lessons from accidents:-

(i) Health Monitoring (detection of deterioration in service):
Particular attention should be paid to health monitoring provisions, inspection schedules, procedures, and rejection criteria. Elementary errors have occurred.

(ii) *Structure in proximity to Drive Shafts:*
Ensure that critical clearances are maintained.

(iii) Lubrication:
Particular attention is required in relation to oil leaks, the inspection and replenishment of grease lubricated components, and cleanliness in oil or grease replenishment.

(iv) Component Replacement:
Ensure correct parts are used and correctly fitted.

(v) Cooler Belt Drives:
Ensure effective inspection and replacement.

(vi) Oil Filter Assemblies:
Ensure torque limits are not exceeded on bolted assemblies.

(vii) Locking of Fasteners:
Ensure locking provisions are correctly used.

3.4 Summarising the lessons from accidents attributed to pilot error/ground crew:-

(i) Overtorquing:
Ensure that incidents of torque exceedance and rotor strikes are reported and correctly actioned. Avoid underestimating torque-time exposure above red line.

(ii) Oil System Warning Lights:
Never ignore warning lights nor accept advice to do so.

4. LESSONS FROM OVERHAUL AND SERVICE OPERATION

In addition to lessons from accidents there is a great deal to be learned from overhaul inspections and service experience that could influence the integrity and reliability of transmissions:-

(i) Condition Monitoring (assessment of condition at overhaul):
Critical components should be carefully inspected for design guidance before scrapping at overhaul (ref.3).

(ii) Corrosion:
Corrosion has been reported (ref.7) as the leading cause of transmission parts rejected at overhaul for a helicopter type operated by the U.S. Navy - 40 per cent of gears and bearings are discarded or re-worked for corrosion. The problem is probably worse for helicopters operating in a sea environment but is by no means restricted to that. Corrosion is a frequently reported problem on gearboxes, even new ones, that have been left in storage for long periods, or spent a long time in

transit. New gearboxes contain corrosion inhibiting oil, but regular oil wetting of all components is necessary to prevent corrosion in storage.

Concern has also been expressed over fretting at gear/shaft bolted interfaces (e.g. ref.8).

A survey of a small sample of bearings (240) from eight gearboxes used in naval, army, and civil operations, showed that of the 72 bearings scrapped at overhaul, 8 per cent of which were due to corrosion and 11 per cent due to fretting.

It is clear that design action is necessary to prevent corrosion of critical gears and bearings, either through the use of truly corrosion- resistant materials, surface treatments, oils with improved corrosion protection, or by using more effective seals and desiccated breathers to prevent ingress of moist air at shut-down.

Problems have also occurred in service and in gearbox production acceptance testing with damage attributed to the surface and sub-surface conditions introduced by corrosion-protection treatments such as phosphating and 'black oxide'. This has led one manufacturer at least to abandon such protective treatments and rely on the protection afforded by corrosion inhibitors in the oil.

(iii) Micropitting:
Other conditions noted at overhaul include micropitting of gear teeth. This is a condition that for a long time was not generally considered to represent a safety hazard, but several incidents in the last decade have caused it to be taken more seriously, both in terms of research aimed at preventing its occurrence, and of closer monitoring in service. It has been found that micropitting can variously remain benign through to overhaul, otherwise can develop into large scale pitting and tooth fracture, or can continue to propagate at small scale resulting in gross erosion of the tooth working flank, leading to possible fracture (ref.9).

(iv) Debris Damage:
In the bearing survey referred to above 8 per cent of the sample showed evidence of craters in raceways formed by rolled-in debris. Incidence of spalling (6 per cent) could have been influenced by such damage.

(v) Sight Glass Staining:
Another condition with serious implications for safety is the staining of oil sight glasses by the oil, giving the impression that oil is present in the gearbox up to or above the 'full' mark, when in fact the oil surface is not visible, and may be below the required level. Recent laboratory tests to reproduce the condition have been only partially successful to date. The tests have included long periods of exposure to temperatures above additive de-composition temperatures in the presence of water, finely divided aluminium and magnesium, and exposure to ultra-violet light. Regular cleaning or replacement of glasses is recommended.

(vi) Human Error:
Hazards of a less 'natural' kind also need to be considered by the designer in the quest for safe gearboxes. One example is given in a recent article (ref.10) where the US Navy has had to check maintenance records and inspect gearboxes that could have been built by a particular mechanic who had the habit of leaving out washers, omitting to lock others, and discarding left- over parts after rebuilding engine accessory drive gearboxes.

5. GEARBOX SAFETY AND RELIABILITY - DESIGN PROBLEMS AND IMPROVED TECHNOLOGIES

Some of the problems facing transmission designers are:-

(i) Fatigue in casings, gears, shafts, and splines, and rolling contact fatigue in gears and bearings.

(ii) Impact loads due to peculiarities in the free-wheels or their activation, and occasional blade strikes.

(iii) Prediction of loads - flight load spectra, cycle frequency, and growth in aircraft gross weight capability.

(iv) Prediction of deflections in drive shafts due to flexure of the airframe, and of gearbox casings.

(v) Establishing materials strength data, especially relative to age and service related degradation.

(vi) Generation of adequate surface separations in gears and bearings operating in the high torque, low speed output stages.

(vii) Contamination control - in assembly, service, and maintenance.

(viii) Joining of complex geometry components to provide adequate stiffness, strength, and freedom from fretting.

(ix) Ingress of moisture, or salt-laden air through natural breathing and through seals, and corrosion prevention generally.

(x) Keeping track of detailed design errors in previous transmissions ('collective memory' - ref.11).

(xi) Having adequate reliability models for all modes of degradation and failure.

(xii) Errors introduced in manufacture, maintenance, and operation, including environmental hazards.

5.1 Bending Fatigue and the Prediction of Loads:

Whilst different terminology is used in the civil and military requirements they both permit the alternative approaches of 'safe life' or 'damage tolerance' relative to fatigue life of rotorcraft structures. The UK civil requirements relating to fatigue life substantiation are reproduced in reference 12, and a strategy for the fatigue substantiation of dynamic components in military helicopters is discussed in reference 13. Figure 1, developed from reference 12,indicates the activities and the problems involved in establishing a safe fatigue life (see also ref.8). Reference 12 contains a similar chart for the damage tolerance substantiation method - this approach being strongly favoured for helicopter structures by the CAA, the Federal Aviation Administration (FAA), and other certification bodies. Whilst the damage tolerance approach has been applied to fixed wing aircraft structures for some time, helicopter transmissions are still being designed on safe-life principles, due to their greater mechanical complexity.

Referring to Figure 1, slight errors or changes in any one of the elements of the process can significantly affect the predicted fatigue life. A study reported in reference 12 involving several US helicopter manufacturers showed an alarmingly wide variation in assumed loads spectra, S/N curve shapes for steels, and reduction factors for scatter, and in their methods of reducing given loads data. It is also reported that one North Sea operator at least spends 90 per cent of flight time at maximum level flight speed compared with less than 40 per cent assumed in design, whilst another was executing seven landings per hour, compared with the declared rate of three per hour.

Operational data is thus essential for all types of helicopters and roles. The UK Ministry of Defence has embarked upon operational data recording (ODR) programmes on several of its helicopters (ref.14), and the CAA have supported and managed ODR programmes on several civil helicopters operating over the North Sea (e.g. ref.15). Results from these programmes could have a major impact on the safety of future helicopters and their transmissions - particularly where direct load measurements are made in critical areas, including torque in both the main and tail transmissions. There would be merit in establishing international agreement on load spectra-cyclic usage relating to different helicopter types, operating roles and geographical areas. Helicopters operating over the North Sea, for example include many different types, manufacturers and countries of origin.

In view of the wide variation in S/N curve shapes for steel coupons referred to above it would be useful to have S/N curve data obtained from full scale gears of various types. Unfortunately little if any such data relating to aircraft quality gear steels and manufacturing processes have been published. S/N data for ground spiral bevel gears in S.82 material (see section 5.5) is therefore presented in figure 2, based on data from tests on a helicopter tail rotor gearbox, from reference 16. Scatter of test points is within expected limits and enables the curve shape and endurance limit characteristics to be established. It would be useful to have such data for modern steels, production processes and finishing standards, and data for other tooth forms.

There has been a substantial increase over the years in the required load factor to be applied to gearbox rig tests to substantiate its strength relative to predicted service loads. Main rotor gearboxes designed in the 1950's for UK military use were tested with a factor of 1.1 on power for three specimens or 1.2 for only one specimen. Experience showed these factors to be inadequate, and discussions between CAA, Royal Aircraft Establishment (RAE), and Westland resulted in the testing factors being raised to 1.4 (one specimen) or 1.3 (four specimens). The relationship between these factors, the S/N curve shape discussed above, and confidence levels is discussed in reference 16. An alternative approach to the derivation of allowable fatigue strength is given in reference 8 where the factor used is the lowest of: mean minus 3 sigma, 0.8 times mean, and bottom of scatter from component tests.

At these very high loads deflections of casings, shafts, and gear teeth can differ significantly from corresponding in-flight loads and it is therefore necessary to apply modified tooth profiles in rig tests to achieve representative contact patterns. Such modifications are too small to affect the strength of the teeth. It may also be necessary to use a non standard lubricant with highest possible load capacity in these rig fatigue tests in order to prevent damage initiating and propagating from the working flank, rather than at the critical 'bending' stress position in the tooth roots. Surface damage can of course occur in service but this possibility is evaluated by endurance tests using the service lubricant at unfactored service loads.

Surface damage occurring in service is easier to detect before it becomes a risk to airworthiness, than fatigue cracks initiated in tooth roots or gear bores, which can propagate relatively quickly without releasing tell-tale debris. However, the advent of effective gear fatigue monitoring techniques (ref.17) offers the possibility of pursuing a damage-tolerant approach to gears in future - i.e. 'inspection-dependent' retirement (ref. 12, 13, 16). This possibility is enhanced by the superior performance of modern steels and production processes in terms of longer life and greatly reduced damage propagation rates (section 5.5). In any event, the combination of usage monitoring, i.e. continuous on-board monitoring of torques, together with effective health monitoring techniques for fatigue and surface distress offers a considerable enhancement of gearbox integrity (see section 5.7).

5.2 Rolling Contact Fatigue:

Rolling contact fatigue (rcf) in bearings and in gear teeth does not exhibit the endurance limit characteristics of bending fatigue and this form of surface degradation is likely to occur at some point in time unless pre-empted by component retirement. The very wide scatter in rcf damage initiation makes retirement an impractical method of control. The scatter can be reduced to an extent and rcf lives increased significantly by improved contamination control and surface separation to reduce the incidence of surface-initiated rcf, but the accepted method of controlling safety in service is by monitoring rcf debris manifestations. Whilst this places the burden of control on the operator this approach is justified by the demonstrably long rcf propagation times from first indications, by the consistency of 'significant' manifestation trends with damage progression, and by the ease with which rcf debris arisings can be assessed. In the writer's experience rcf debris from hardened steel gears or bearings is typically 'thin flake' in nature and the leading dimension generally greater than 100 micro-metres on release. Simple magnetic plugs or 'chip collectors' have been used effectively for many years by operators both civil and military, with one notable exception that appears in the list of transmission-related accidents. Health monitoring methods capable of monitoring only very small size debris, such as spectrometric oil analysis, however quickly the results are obtained and however presented, seldom give consistent and reliable indications of rolling contact fatigue in gearboxes in advance of chip collectors in the writer's experience. Improved methods of wear debris monitoring to ease the operators task are reviewed in section (5.7).

Debris released by micropitting tends to be very much smaller and more difficult to monitor with chip collectors, but the process has no significance to safety or reliability unless it progresses to the macro form of pitting, or continues unchecked until the tooth profile is significantly modified, then becoming detectable by vibration analysis (ref.9). Micropitting can be delayed or prevented altogether by proper control of surface finish - by avoiding surface etching processes and by employing processes which tend to smooth out the grinding asperities and induce compressive stress (e.g. Abral), or surface deposition treatments having smooth, crack-resistant properties (e.g. Thin Dense Chrome).

5.3 Corrosion and Fretting:

Magnesium alloy castings have been normal practice for helicopter gearbox casings for many years, having excellent strength/weight and stiffness/weight ratios, and damping characteristics. Effective corrosion protection is afforded by chromate conversion coatings, and epoxy resin sealers. Diligence is required in the speedy repair of localised damage to the protective coating in service. Casings in carbon fibre re-inforced plastic (CFRP) have been tested (ref.7) and demonstrated weight and stiffness advantages, but have yet to be adopted in production.

Strength properties are normally the over-riding concern when selecting steels for bearings and gears, and non-corrosion resistant materials result. Anti-corrosion treatments used on gears for many years, principally phosphating and black oxide alkaline coating, have proved effective in preventing corrosion but have exhibited harmful side effects. The nodular surface finish produced by the phosphate acid etching process appears difficult to control and has led to scuffing and to micropitting. Black oxide has been shown to enhance crack initiation and propagation. Recent transmission designs have therefore featured high strength steels used in the bright condition, and reliance placed upon the lubricating oil to prevent corrosion in service, and inhibiting oils and good housekeeping in storage. Some of the commonly used lubricants are better than others for corrosion prevention. Overhaul inspection records reported in ref.7 show a ratio of 10 to 1 for gears and 31 to 1 for bearings in the percentage of components corroded in gearboxes using two different oil specifications. The recent development of Thin Dense Chrome (TDC) treatment for gears and bearings promises a satisfactory solution for future designs, free from other drawbacks (section 5.5).

Several accidents and reliability problems have been attributed to fretting corrosion. Highly loaded spiral beval gears clamped or bolted to the shaft, and bearing/spacer assemblies relying on nut clamping loads to prevent rotation are common problem areas. Boeing Vertol have demonstrated (ref.7) the manufacture of a fully integrated gear/bearing/shaft design. The much higher levels of safety and reliability afforded by such designs will probably more than offset the increased cost of the component. The significant reduction in number of parts must also tend to reduce first cost and weight.

Intrascope provisions are recommended to aid inspection of internal components for corrosion and fretting, avoiding the need for partial disassembly.

5.4 Lubrication Systems and Contamination Control:

In addition to the detailed design points noted under 'lessons from accidents', developments in oil filter technology and gearbox component cleaning facilities offer significant improvement in life and reliability (ref.17). Gearboxes with recirculatory oil systems can benefit from the new fibre matrix materials, and from location of the by-pass valve at entry to the filter. The former permits filtration down to very small particle sizes without significant pressure drop penalty, and the latter prevents scouring of filtrate into the oil jets on start-up. Reference 17 describes an application in which conventional filters of approximately 40 micro-metres <u>nominal</u> rating are replaced by filters of 3 micro-metres <u>absolute</u> rating within a similar size envelope. Filtration at this level prevents the recirculation of particles capable of causing debris indentations in bearing raceways and premature rolling contact fatigue.

Vapour honing facilities for cleaning components immediately prior to assembly have proved effective in removing adhered and semi-adhered debris left on the surfaces after manufacture, inspection, storage, and handling - including ferrous oxide particulates used in crack detection inks (ref.17). By use of these facilities build cleanliness standards equivalent to 3 micro-metres absolute filtration can be achieved prior to first turn-over for splash lubricated and pressure lubricated gearboxes. Gearboxes cleaned by this process, and fitted with 3 micro-metres absolute filters have

only recently entered service, and it will take some years to quantify the benefits in terms of improved reliability and safety.

Diligent cleaning of oil passageways between the filter and oil jets is essential to gearbox integrity. Analytical and experimental studies of oil jet configurations reported in reference 18 permit more effective use of the oil pump and available cooling capacity.

5.5 Materials and Processing Developments:

Developments in casing materials have much to contribute to improved safety principally through corrosion resistance and increased damage tolerance - several of the accidents reviewed involved fracture of gearbox feet and attachments to the structure. Reference 7 describes the manufacture and testing of a main rotor gearbox casing in CFRP composite material offering significant weight savings in addition to the safety advantage. CFRP also has significant weight, reliability, and safety advantages over aluminium alloy for drive shafts in that the greater stiffness permits longer bearing spans, and therefore fewer support bearings and couplings. Boeing Vertol have also applied this technology to a main rotor shaft. The more recent development of metal matrix composites offers even greater enhancements in terms of damage tolerance, stiffness, and temperature capability. An alternative approach is that of fabricated casings in steel or titanium (e.g. refs 19, 20).

The primary requirements for gear and bearing steels for helicopter transmissions are suggested to be:

(i) High fatigue strength in both core and surface material.
(ii) High fracture toughness to withstand impact loads.
(iii) Slow crack propagation rates.
(iv) High resistance to surface damage and corrosion.
(v) Suitable for integral gear/shaft/bearing components.

Most of the failed gears and bearings referred to in Appendix 3 were manufactured in air-melt steels - SAE 52100/En31 (1.5 Cr, 1.0C) through- hardening type for bearings, and case-hardening type for gears and integral bearing raceways - S.82 (4Ni, 1.2Cr, 0.25 Mo, 0.16C) in the UK and AISI 9310 (3.5Ni, 1.2Cr, 0.1Mo, 0.1C) in the USA.

Problems with the control of inclusion size and dispersion caused considerable variability in fatigue strength, both in bending and in rolling contact fatigue. Introduction of vacuum degassing facilities led to greatly improved versions of the same basic formulations becoming available for helicopter applications in the early 1960's followed by Vacuum Arc Remelting (VAR) of the air melt steels for ingots up to 200mm, and Electro- Flux Remelting (EFR) for larger ingots in the late 1960's, the VAR version of S.82 being designated S.156. US Army experience in operating helicopters in Vietnam led to the requirement in the late 1960's for steels with higher temperature capability to provide greater tolerance to loss of oil. Molydenum tool steel (M50 - 4.2 Mo, 4.1 Cr, 1.0V, 0.8C) was available in the USA in VAR form with a fatigue life four times greater than the traditional air-melt bearing steel at normal temperatures and very much greater at elevated temperatures. In the early 1970's Vacuum Induction Melted (VIM) M50 became available, replacing the air-melt version and VIM VAR M50 demonstrated up to forty fold rcf life improvement factors in service over air-melt SAE 52100. Current helicopter transmissions tend to contain a combination of vacuum degassed SAE 52100 and VIMVAR M50 bearings with silver-plated steel cages, and S156 or AISI 9310 gears (ref.21). Several high temperature case carburising steels suitable for bearings and gears have been developed in the USA including X-2M, CBS 1000M, M50 NiL and EX53 (refs 22, 23, 24), and Firths RBD (10W, 3Cr, 0.4V, 0.2C) has been used in UK gas turbine applications for many years for hot-end bearings,and for locations producing high hoop stresses in the rings. M50 NiL (4Mo, 4Cr, 1.3V, 3.5 Ni, 0.1C) is a more recent development used in similar applications, and promises to be an ideal material for integral bearing/gear/shaft components. In addition to greater hoop-stress capability M50 NiL has exhibited significantly better rolling contact fatigue resistance than M50. The greater hoop stress or bending stress capabilities of M50 NiL and X-2M is due both to the lower carbon content in the core than M50 and to the large residual compressive stresses readily developed in the case - often capable of stopping crack propagation (ref. 22).

Nitriding steels have been used for helicopter gears, particularly in France, but their potential in future applications is limited by their shallow case depths, lower core strengths, and the need for effective nitride layer removal. Trials are continuing on ways to overcome these problems (ref.24). A further enhancement of steel cleanliness and hence fatigue properties is afforded by the recent introduction of ladle steelmaking techniques, and the more detailed understanding of inclusions (ref.21).

Recent developments in surface treatments offer improved scuff resistance, corrosion resistance, and further enhancement of resistance to surface- initiated rolling contact fatigue. These include very thin coatings of hard material produced by ion-implantation or Poly Vapour Deposition (ref.21) or by electro-deposition - e.g. Thin Dense Chrome (TDC) (ref.25).
Improved gear tooth 'bending' fatigue performance and rolling contact fatigue resistance is also afforded by modern finishing processes after grinding - e.g. controlled shot-peening of roots, and Abral treatment (vibrating components in zinc pellet medium), which induce residual compressive stresses in surface material and improve surface finish.

5.6 Bearing Technology:

In addition to the substantial improvement in service life afforded by improved steels, surface treatments, and contamination control, the following factors contribute to improved integrity of bearings:-

* Improved cage design and materials.
* Lubrication via bores of shafts.
* Improved performance data relating to materials and lubrication (e.g. ref.26).
* Application of life prediction based on:
  (a) experience - individual locations with difficult duty or environment, and
  (b) system reliability models (e.g. ref.23).

5.7 Health and Usage Monitoring (HUM):

Technology advances have been established over the last decade which offer considerable improvements in flight safety:

(i) On-board data processing and display systems (e.g. refs 15, 17, 27, 28) providing:

* continuous, accurate measurement of torques
* advanced vibration analysis for fracture modes
* continuous wear debris monitoring for surface degradation
* remote monitoring of oil level for maintenance aid and leak detection
* advanced rotor track and balance facilities to minimise rotor-induced loads

(ii) Improved ground-based techniques for:

* oil analysis - for monitoring corrosion, fretting, and micro-pitting
* monitor filter to aid ground inspection of non-ferrous wear problems
* intrascope provisions for confirmation of surface damage indications, and inspection for corrosion

An extensive review of health monitoring techniques relevant to helicopters including transmissions is given in reference 29.

HUM system development effort should be targeted at advanced warnings for maintenance intervention. However, the observation in Appendix 3, that more than 60 per cent of serious accidents occurred during cruise flight, underlines the case for providing 'last-ditch' cockpit warnings in addition.

The importance of such developments to airworthiness is appreciated by the airworthiness authorities in the UK (e.g. refs 1, 2, 16). The CAA is encouraging the provision of health and usage monitoring to new transmissions and rotor systems in particular by incentives - by allowing airworthiness credit relative to the recently tightened safety requirements (BCAR Paper No. G811-ref.2). The CAA, FAA, and other civil airworthiness authorities are in the process of introducing mandatory requirements for Flight Data Recorders (FDRs) to be fitted to transport helicopters above 2700Kg maximum weight in 1991. Whilst these will be crash-protected recorders for accident investigation purposes, the requirement will increase the incentives for operators and constructors to add HUM processors/recorders by interfacing them with the Flight Data Acquisition Unit required for the FDR thereby saving cost and weight. Input of HUM processed data to the FDR could also add considerably to the usefulness of the FDR.

The UK Ministry of Defence is in the process of making the provision of advanced monitoring techniques mandatory on new helicopter transmissions, using the design requirements document, DEF-STAN 00-970 Vol. 2, as the means of achieving this (ref.16).

A welcome development in the USA is the recent decision by the Society of Automotive Engineers E-32 committee (engine monitoring) to extend its work to cover helicopter transmissions.

One concern in the widespread application of HUM systems, to operators, constructors, and airworthiness authorities alike will be the need to have speedy access to information from accident investigations, sufficient to permit early refinement of health or usage monitoring algorithms universally. This additional task on accident investigation authorities, whilst representing possibly an additional burden, is very important to the achievement of improved safety levels. Advanced HUM technology provides a greatly improved 'measuring stick' for component integrity, with tentative 'markings' relating to pre-conceived modes of degradation and rejection limits, often based upon component tests in a non-service environment. The usefulness of HUM systems will improve as real degradation modes and rejection limits relating to service experience are incorporated. An attempt has been made in Appendix 3, table 6 to indicate the type of monitoring technology appropriate to the cause of each transmission related accident for which sufficient information is given in the Accident Summary narratives. This leads to encouragement in the sense that appropriate monitoring techniques are gaining maturity, but also reveals that much more detail than that given in many Accident Investigation Reports will be required in future to permit refinement of algorithms and limits.

5.8 Safety Assessment:

In the past the assessment of safety aspects of transmission designs has been performed by designers in parallel with weight, producibility, and cost assessments without formalised requirements.

The British Civil Airworthiness Requirements (BCAR) for rotorcraft now call for a fault analysis to be carried out on the rotor and transmission systems. The fault analysis is to be of the effects of a single failure but is extended to multiple failures if the first failure would not be detected during normal operation or if the first failure could lead to other failures before the first failure is rectified (ref.3).

The objectives of the failure analysis are:

* Identify failure modes and likely effects.
* Re-design those components predicted to exhibit serious failure modes.
* Assess probability of the failures and set service inspection intervals accordingly.
* Determine appropriate health monitoring techniques.
* Identify critical components requiring special controls in design and manufacture.

An example of such analysis is given in (ref.3).

Deeper discussion of alternative formalised approaches to failure analysis - Failure Modes and Effects Analysis (FMEA); Failure Modes, Effects and Criticality Analysis (FMECA); and Fault-Tree

Analysis (FTA) are given in reference 30. FMEA is a study in cause and effect; FTA is the reverse procedure - in which the final effect is taken as the top event and is traced back to the possible primary causes of that failure mode. These are best performed by computer analysis, and programmes are now commercially available for this purpose.
Statistical models of component/assembly/vehicle behaviour in service - reliability models - are essential aids to the safety assessment. These involve statistical distribution curves, the most appropriate to high reliability transmissions being the 'three parameter Weibull' and 'Modified Weibull' empirically based models, and a good example of the application of these to helicopter gearbox failure data is given in reference 30. A more detailed discussion of life statistics in general, and component life predictions in particular is given in Chapter II.

5.9 Transmission Configuration:

The primary configuration of the complete transmission system is dictated by the rotor configuration, and the number and location of engines. In the accident data analysis in Appendix 3, insufficient data was available to determine whether rotor configuration or associated transmission complexity had a significant influence on safety. However, the consequences of loss of tandem rotor phase separation - very few survivors in the civil transport data analysed - are such as to justify the greatest possible design attention to all components involved in rotor synchronisation.

Alternatives to the conventional tail rotor for reacting the main rotor torque (e.g. ref.6), have potentially enormous safety benefits, noting the significant number of tail rotor strikes, passenger collisions, and drive shaft/transmission/rotor control system failures in reference 5.

Regarding alternative gear configurations within main rotor gearboxes, the majority of designs in the past have employed planetary gearing in the high torque output stages. Of the 13 helicopter types included in the accident data survey in Appendix 3, 11 employ planetary gears, 7 containing two planetary stages. Whilst this compact form of gearing permits efficient transmission of power at the high torque end of the gearbox, they provide potential airworthiness concerns including:

* Power combining stage gearing in multi-engined helicopters has to be 'upstream' of the output stage, resulting in more gears and bearings in the critical single path driving the rotor.

* The common practice of using the bores of planet gears as the outer raceway of the planet supporting bearing produces a complex stress field in the planet gear which potentially enhances fatigue crack initiation and propagation rate.

* Increased likelihood of rolling contact fatigue damage initiation by debris from mast bearing spalling.

* Fast damage propagation through debris entrapment in compact, crowded gear assemblies, particularly with two-stage planetaries.

'Parallel shaft' or 'branched' gearing arrangements as used in the Sikorsky S76, the Westland 30/Lynx, and the MBB-Kawasaki BK117 avoid these safety hazards, and offer potential weight advantages (ref.20). Having said that, only two of the accidents reviewed in Appendix 3 that have been fully investigated and reported have involved gearbox failures in the single output load-path, one of these due to a spiral bevel gear failure and the other in a two-stage planetary gear - due to a failure of a second stage planetary bearing that could have been prevented by proper use of the monitoring provisions and manufacturers recommendations. A planetary gear failure in a military helicopter was reported in detail in reference 3 with the recommendation that planet gears should not run directly on bearing rolling elements. A planet gear failure in a smaller helicopter was reported in reference 4. Examples of several gear arrangements in common use are shown in reference 31.

Regarding tooth geometries, service experience has been gained with conformal tooth forms (Westland) and test experience with high contact ratio (HCR) involutes produced by symmetric profile modification (Boeing Vertol), and HCR buttressed involutes produced by increasing the pressure angle on the normally non-loaded flanks (Sikorsky), but the involute tooth form with appropriate profile relief continues to be used universally by all manufacturers. Whilst conformal

tooth forms, and possibly the buttressed HCR involutes (ref.19) may have some integrity advantages, and symmetric HCR involutes the reverse (ref.32), the over-riding issue as far as integrity is concerned is tooth contact pattern control - the design and manufacturing procedures required to ensure that tooth contact patterns remain within the well supported area of the tooth working flank particularly at high load conditions.

5.10 Analysis of Stresses, Distortions, and Surface Separations:

Several of the accidents investigated in Appendix 3 were attributed to overload, distortions and misalignments, or underdesign.

The last decade has witnessed the transformation of computer based analysis (e.g. Finite Element Analysis) of complex problems, from a post-design activity undertaken in parallel with development (often chasing problems identified during tests) to a routine preliminary design activity upon which detailed design is heavily dependent. This includes complete gearbox system static and dynamic models, and highly detailed models of individual components such as casings, gears, and splined quill shafts (e.g. ref.33). Moving mode-shape displays of gearbox dynamic behaviour are ideal for preventing resonance conditions and dynamic problems which often reveal themselves during development testing or in service operation. The availability of such facilities permits the use of complex components such as the integral gear/shaft/bearing component described in reference 7 which would otherwise be too costly to develop.

Realistic static and dynamic models of the complete rotorcraft are nowadays an essential part of the design process, and in these the gearboxes are an integral part of the airframe model, and conversely the influence of static and dynamic behaviour of the supporting structure can be included in gearbox and shafting models.

Significant advances have also been made in the analysis of gear tooth separation i.e. lubricant films and surface finish characteristics. Reference 34 summarises developments relating to scuffing, micro-elastohydrodynamic lubrication, and to film thickness predictions in gears, including those with conformal tooth forms.

5.11 Computer Data Bases and Expert Systems Technology:

The routine use of Computer Aided Draughting facilities integrated with Computer Aided Manufacture in transmission design has resulted in the elimination of many of the errors that occurred previously between the original design scheme and the manufactured component. These facilities have proved invaluable in multi-national collaborative projects which are increasingly becoming the norm with military aircraft, helicopter, and engine projects.

Safety is also enhanced through better control of the designer's use of materials strength data, manufacturing process data, and other engineering standards which are updated from time to time. There is a need to use materials strength data pertaining to age and environmental related effects in service. It may take many years to generate such data, but a computer data base is considered essential for its storage and implementation.

'Expert Systems' or 'Knowledge Based (KB) Systems' are becoming sufficiently well developed to offer a significant contribution to improving airworthiness, including the following areas:-

(i) Conceptual Design - Intelligent Computer Aided Design:
A 'Knowledge based system' interfaced to CAD/CAM files and all data bases used by a designer can permit automation of the design process to the extent that design alternatives can be evaluated in depth relative to safety, weight, cost, etc.

(ii) Lessons from Errors and Accidents - 'Collective Memory':
Both conceptual and detailed design could be improved by a KB system which encapsulates and orders lessons from experience, including feed back from overhaul (Condition Monitoring) and accident investigation, otherwise made difficult by the long time-span of a helicopter design, development, and service life, and the large number of repositories of knowledge.

(iii) Interpretation of Failures and Wear:
Interpretation of wear and failure conditions observed in used transmission components is highly complex (e.g. ref.35) and expert experience is in short supply. Encapsulation of this expertise in an interactive system with graphical and textual descriptions would be an invaluable aid to improving gearbox integrity.

(iv) Interpretation of Health Monitoring Outputs:
Interpretation of service data in terms of combinations of diagnostic techniques and of pattern recognition in techniques with complex outputs also requires expertise, which is scarce, and a support system having 'learning' capability would be invaluable (e.g. ref.36). Also required is the generation of success/failure indication matrices for each technique, condition, and rejection limit for actual arisings.

5.12 Manufacture, Qualification, and Maintenance Control:

Transmission design for safety does not stop at the definition of components, assemblies and tolerances but extends to control of manufacture and maintenance in respect of:-

(i) Definition of manufacturing, assembly and inspection processes through call up of appropriate company standards.

(ii) Identification of critical parts requiring appropriate quality control procedures.

(iii) Definition of component cleaning procedures prior to assembly.

(iv) Definition of design qualification or substantiation test requirements, to include:-

- (a) fatigue tests at overload (minimum: 4 tests at 1.3 load factor),
- (b) endurance and environmental tests including temperature,
- (c) vibration and deflection survey, and comparison with predictions,
- (d) in-service health and usage monitoring provisions to be applied to all tests, to demonstrate 'visibility' of all components which are dependent upon HUM to meet the safety requirements, and the validity of retirement and caution indications,
- (e) demonstrate the effectiveness of any health and usage monitoring provisions proposed as alternatives to those which may be mandated or established practice,

(v) Definition of production build standard, acceptance test monitoring techniques and acceptance criteria (e.g. tooth meshing patterns and vibration signatures).

(vi) Identification of life limited parts and their lives.

(vii) Definition of maintenance tasks, schedules, procedures, and equipment for inclusion in the Maintenance Manuals.

(viii) Justification for initial Time Between Overhaul (TBO), and definition of health and usage monitoring techniques/criteria to be used in 'on-condition' maintenance programmes, noting the CAA position (ref.2) 'that overhaul periods and sampling intervals will be restrictive unless effective health monitoring methods are practiced to control hazardous failures which are not directly time/cycle related'.

(ix) Overhaul and defect inspection (condition monitoring) - ensure effective recording/feedback system and design involvement in inspections (e.g. ref.37).

6. CONCLUSIONS

(i) The accident data analysed showed that transmissions accounted for 22 per cent of potentially airworthiness related accidents in civil helicopters over 4550Kg gross weight. Design action is therefore required to reduce the incidence of design and manufacture related failures in future transmissions and to assist in the reduction of maintenance-induced failures.

(ii) Sufficient information is given in many of the narrative reports in the CAA World Helicopter Accident Summary to permit qualitative assessment of the likely safety benefits of technology improvements relating to transmission components, to design and manufacturing facilities, and assessment of health and usage monitoring requirements in general terms.

(iii) Confidence can be placed in design and technology advancements over the last decade to improve the safety and reliability of transmissions considerably, including:

* Advanced computer-aided design/analysis - gearbox applications.
* Measured operational spectra and loads.
* Greatly improved steels permitting better fatigue performance and integral gear/shaft/bearing designs.
* S/N curve refinement.
* Computer-aided manufacture, improved accuracy, improved finishing techniques and component cleaning facilities.
* Improved lubricants, fine filtration, and emergency lubrication systems.
* Advanced health and usage monitoring systems.
* Expert systems technology to encapsulate and retain the 'collective memory' relating design to service experience, including the data available from overhaul, incidents, and accidents.

7. RECOMMENDATIONS

In addition to specific recommendations made within the body of the paper, the following more general suggestions are made:

(i) Ensure, in all future transmission designs, that a Fault Tree Analysis is performed at the preliminary design stage, together with a Failure Modes, Effects, and Criticality Analysis. This should clearly identify those components and failure modes which require health or load exposure monitoring to achieve the required levels of safety for the complete systems, and should indicate the precise form of monitoring, implementation, and sampling frequency proposed to satisfy this need.

(ii) Create a 'Degradation and Failure' data base to facilitate retrieval of:-

* Materials strength data related to in-service performance.
* Wear, corrosion, and degradation found in overhaul inspections.
* Lessons from accidents, by system, component, and cause (design, manufacture, maintenance, or operation).
* Usage monitoring performance - correlation with component condition.
* Health monitoring system performance - correlation with condition in terms of success, failure to indicate, false indications, and implementation problems.

(iii) Appraise original safety analysis against degradation and failure data at appropriate intervals (e.g. when extension of service life is sought).

(iv) Remove transmissions for overhaul 'on-condition' after conservative sample inspection programme, i.e. remove for overhaul on the basis of usage monitoring and health monitoring limits plus finite lives where necessary.

(v) Test, to failure, critical components that are life-expired or otherwise rejected from service, in order to assess the effectiveness of health and usage monitoring techniques and algorithms with 'naturally produced' failures.

(vi) Airworthiness authorities, accident investigators, operators, and manufacturers, all have a contribution to make to the data bank recommended in (ii), and would derive benefit from it. It is therefore recommended that airworthiness authorities, government agencies, commercial and military operators, and manufacturers' associations should endeavour to set up a common data bank covering their interests.

(vii) In conjunction with (vi) or otherwise, accident investigation organisations should assist in the early refinement of health and usage monitoring techniques, algorithms, and rejection criteria both in specific incidents, and in general application.

(viii) The scope of the above recommendations should be extended to cover rotor systems and engines, in view of their significant contributions to accidents in the data analysed.

8. REFERENCES

1. CAA *Report of the Helicopter Airworthiness Review Panel,* CAP.491, Civil Aviation Authority, London. June 1984

2. WITHAM, J.E. Safety standards for helicopter and engine transmissions, paper 1, I.Mech.E. seminar *Pushing back the frontiers of failure in aerospace transmissions,* London, December 1986

3. VINALL, P.D. Airworthiness of helicopter transmissions, paper 1, R.Ae.S. symposium *Helicopter transmissions,* London, February 1980

4. CAMPBELL, G.S. and LAHEY, R.T.C. *A survey of serious aircraft accidents involving fatigue fracture,* Vol.2. NAE-AN-8, National Research Council, Canada, April 1983

5. CAA *World helicopter accident summary,* issue CA8, Civil Aviation Authority, London 1988

6. WARWICK, G. Notar makes the grade. *Flight International,* p.26, 27th February 1988

7. DRAGO, R.J. and LENSKI, J.W.Jr. Special power train requirements for the next generation of rotary wing aircraft, paper 5, CP-369, AGARD conference: *Gears and power transmission systems for helicopters and turboprops,* Lisbon, Portugal. October 1984

8. ALBRECHT, C. Designing safety into helicopter transmissions, American Helicopter Society 37th Forum, New Orleans, Louisiana, USA. May 1981

9. ASTRIDGE, D.G. Application of vibration analysis to the health monitoring of high-duty gearboxes, paper 2b, PEMEC conference on *Design engineering,* Birmingham, October 1981

10. U.S. Navy to check J52 engines. *Flight International,* p.13, 16th January 1988

11. MEEKCOMS, K. The origins and development of military airworthiness, paper 2, I.Mech.E. seminar *Safety in aircraft engineering,* January 1986

12. BRISTOW, J.W. Helicopter fatigue - the changing scene, paper 17, American Helicopter Society, *Helicopter fatigue specialists meeting,* St. Louis, USA, October 1984.

13. CARDRICK, A. A strategy for the fatigue substantiation of helicopter dynamic components, paper 2, R.Ae.S. symposium *Helicopter airworthiness,* London, December 1984

14. PEARSON, N.G. and WEST, C.N. Operational load measurement programmes and results, paper 5, I.Mech.E. seminar *Health and usage monitoring in helicopter mechanical systems,* Yeovil, April 1986

15. COGAN, M.E. Helicopter health monitoring - an operator's experience, paper S/606/2, I.Mech.E. conference *Aerotech 87,* Birmingham, October 1987

16. CANSDALE, R. and TIGWELL M.G. Fatigue factors as applied to the testing of helicopter gearboxes, paper 2, I.Mech.E. seminar *Testing of aerospace transmissions,* Derby, June 1987

17. ASTRIDGE, D.G. Improving the integrity, life, and reliability of helicopter gearboxes. paper 5, I.Mech.E. seminar *Pushing back the frontiers of failure in aerospace transmissions.* London, December 1986

18. AKIN, L.S. and TOWNSEND, D.P. Lubricant jet flow phenomena in spur and helical gears with modified addendums - for radially directed individual jets, US Army *AVSCOM Technical Report* 88-C-034, April 1989

19. MANCINI, J.H. An overview of advancements in helicopter transmission design, P109-122, *Advanced power transmission technology,* Cleveland, Ohio, USA. NASA Conference Publication 2210, June 1981

20. COCKING, H. The design of an advanced engineering gearbox, paper 85, *tenth European rotorcraft forum,* The Hague, The Netherlands, August 1984

21. HAMPSHIRE, M. Bearing material advances, paper 4, I.Mech.E. seminar *Pushing back the frontiers of failure in aerospace transmissions,* London, December 1986

22. AVERBACH, B.L. et al. Fatigue crack propagation in carburised high alloy bearing steels/carburised X-2M Steel, *Metallurgical transactions* A Vol 16A, July 1985, p.1253-1271.

23. BRADDOCK, C.E. and BATTLES, R.A. Design of an advanced 500-hp helicopter transmission, p.123-139, *Advanced power transmission technology,* Cleveland, Ohio, USA, NASA Conference Publication 2210, June 1981

24. DAVIES, D.P. and GITTOS, B.C. Gear steels for future helicopter transmissions, paper 7, I.Mech.E. seminar *Design of aerospace transmissions,* Taunton, March 1988

25. WASKIEWICZ, W.P. Extending bearing life in off-highway equipment, Society of Automotive Engineers, paper 841124, 1984

26. BAMBERGER, E.N. et al. Life adjustment factors for ball and roller bearings, ASME *Engineering design guide,* September 1971

27. ROE, J.D. The EH101 health and usage monitoring system - from instigation to implementation, paper S/606/4, I.Mech.E. conference *Aerotech 87,* Birmingham, October 1987

28. ASTRIDGE, D.G. and COLLIER-MARSH, M.E. Operational experience with advanced transmission health monitoring techniques on the Westland 30 helicopter, paper 48, *eleventh European rotorcraft forum,* London, September 1985

29. CAA *Report of the helicopter health monitoring working party,* paper 85012, Civil Aviation Authority, London, August 1985

30. CARTER, A.D.S. *Mechanical reliability,* 2nd edition, p.151, ISBN 0-333- 40587, Macmillan 1986

31. WHITE, G. A 2400kW lightweight helicopter transmission with split torque gear trains, ASME 84-DET-91. October 1984

32. COY, J.J. Geared power transmission technology, NASA conference publication 2210, AVRADCOM report 82-C-16, symposium *Advanced power transmission technology,* Cleveland, Ohio, USA, June 1981

33. REES, D.M.D. Advanced design facilities, paper 4, I.Mech.E. seminar *Design of aerospace transmissions,* Taunton, March 1988

34. SNIDLE, R.W. Some aspects of gear tribology, paper 6, I.Mech.E. seminar *Design of aerospace transmissions,* Taunton, March 1988

35. SHOTTER, B.A. The observation and interpretation of gear tooth failures, paper 31 AGARD conference: *Gears and power transmissions systems for helicopters and turboprops*, CP-369, Lisbon, Portugal, October 1984

36. STEWART, R.M. Machinery management based on the application of modern diagnostics, control theory, and artificial intelligence, conference: *Condition monitoring 84*, University College, Swansea, April 1984

37. ROCKWOOD, J.D. The 214ST lead-the-fleet program, American Helicopter Society *Rotary wing propulsion system specialists meeting*, Williamsburg VA, USA, November 1986

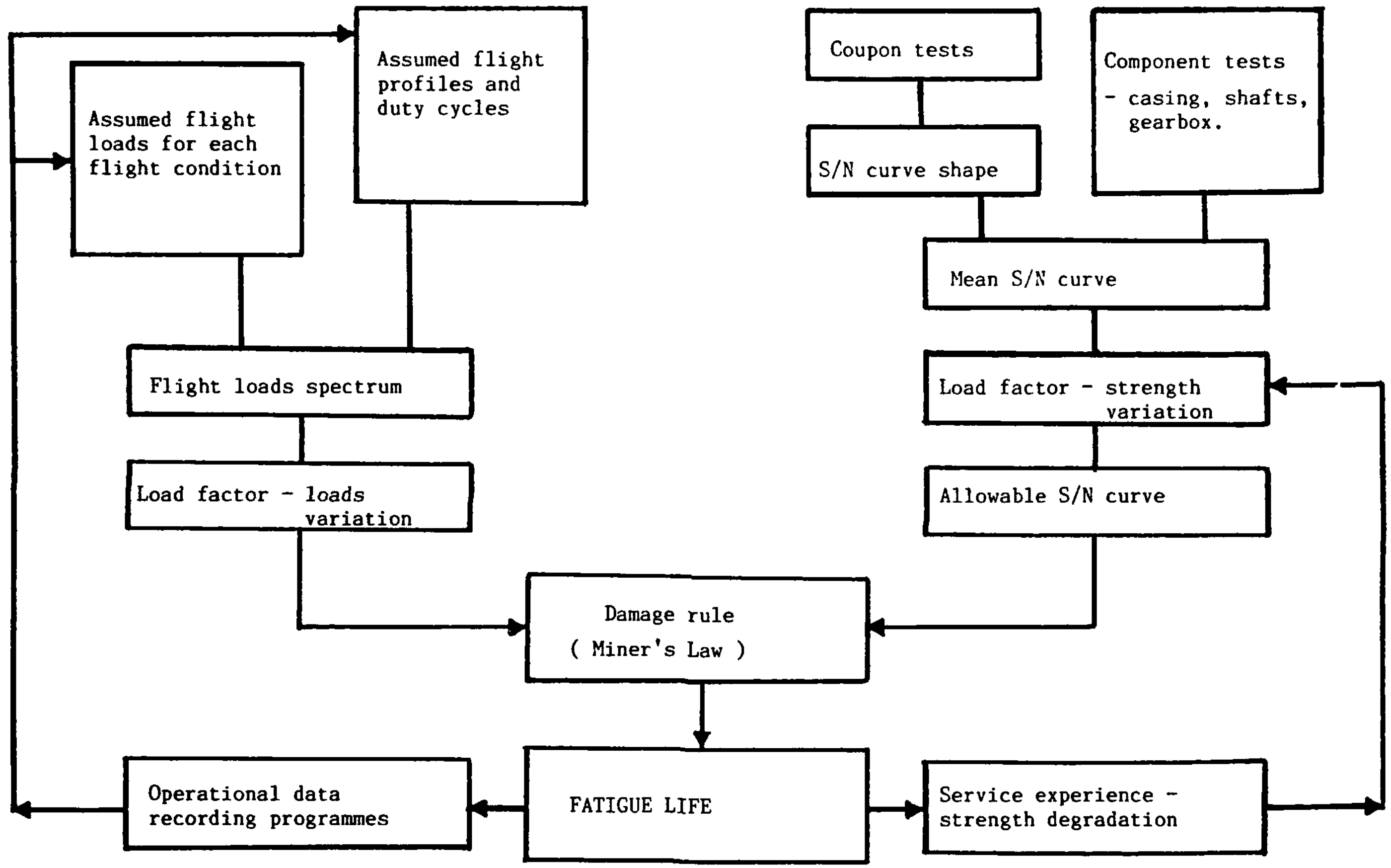

Fig. 1 Helicopter gearbox fatigue life substantiation methodology

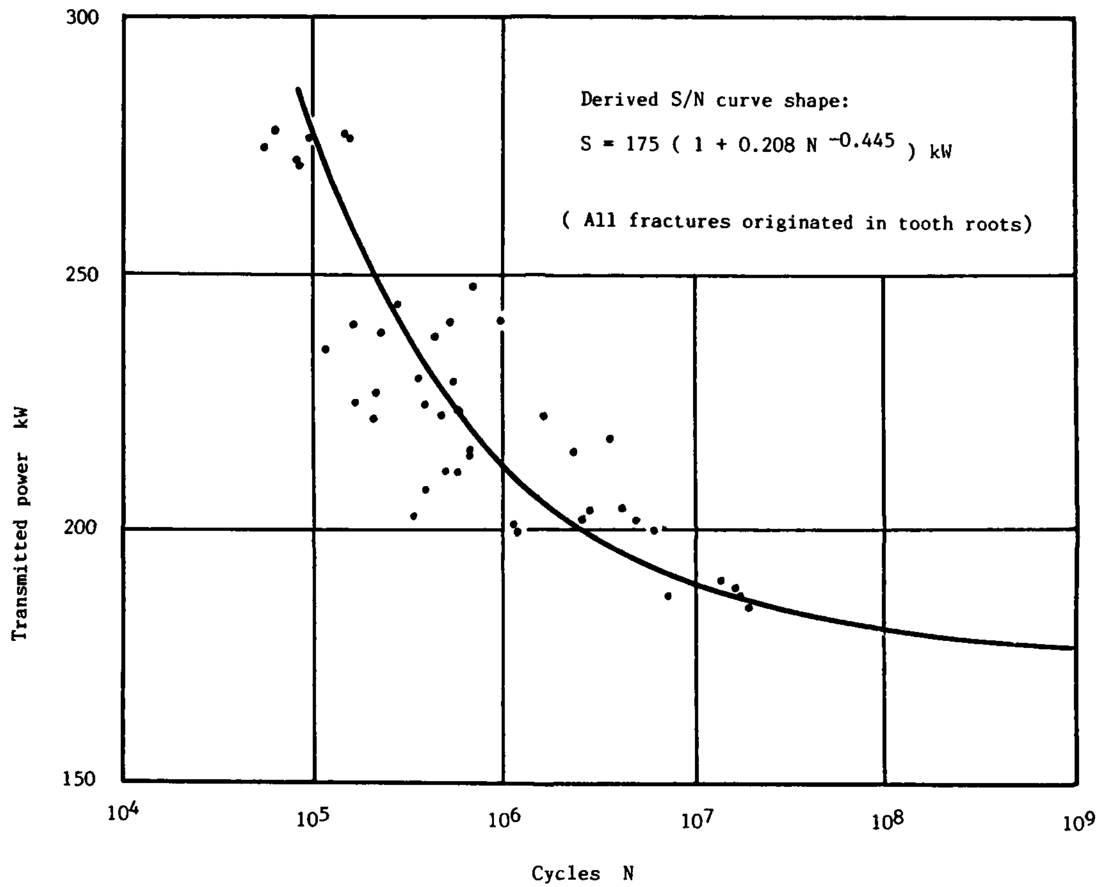

Fig. 2 Fatigue S/N curve for ground spiral bevel gears in S 82 Steel, obtained from helicopter tail rotor gearbox tests to failure.

APPENDIX 1

TRANSMISSION SYSTEM

The Transmission System includes any part necessary to transmit power from the engines to the rotor hubs. This includes gearboxes, shafting, universal joints, couplings, rotor brake asemblies, clutches, supporting bearings for shafting, any attendant accessory pads or drives, and any cooling fans that are a part of, attached to, or mounted on the Transmission System.

APPENDIX 2

AIRCRAFT ACCIDENT

An aircraft accident is defined in the Standards and Recommended Practices - Aircraft Accident Inquiry - Annex 13, of ICAO, as follows:

'An occurrence associated with the operation of an aircraft which takes place between the time any person boards the aircraft with the intention of flight until such time as all such persons have disembarked, in which:

(a) any person suffers death or serious injury as a result of being in or upon the aircraft or by direct contact with the aircraft or anything attached thereto, or

(b) the aircraft receives substantial damage, or

(c) the aircraft is missing or is completely inaccessible.'

APPENDIX 3

TRANSMISSIONS CONTRIBUTION TO ACCIDENTS -

CIVIL TRANSPORT HELICOPTERS GREATER THAN 4550Kg 1956-86

Data Source: CAA World Helicopter Accidents Summary, Issue CA8,
Civil Aviation Authority, London. 1988

Summary:
The CAA Summary provides an analysis of accidents (defined in Appendix 2) by helicopter type, but no analysis relative to component cause. It does however, provide narrative for each entry, many of which are brief and inconclusive, due in various degrees to the wreckage being irrecoverable, investigations incomplete, and to inadequate reporting of cause in early reports, even where the primary component is identified. However, many of the narratives do report the conclusions of investigations and are quite specific about the cause. Analysis of the usable narrative reports is summarized in Table 2, which shows for each of 13 helicopter types, the total number of accidents, those due to maintenance, those due to design or manufacture, and others where the cause is unknown. Whilst the aircraft types are identified in the CAA Summary, they are given a code-letter identification in this study in order to encourage wider debate.

Of the 361 accidents relating to the 13 transport helicopters studied, 151 or 42 per cent of the total were related to pilot, ground staff, or environmental hazard. A similar number (143) of accidents did not have a cause attributed to them, so potentially these could be considered to be due to maintenance, design or manufacture - in the majority of cases the failed component was identified. 22 of the total accidents (6 per cent) were attributed to maintenance and 46 accidents (13 per cent) were attributed to design or manufacture.

Transmission related accidents are shown in brackets and accounted for 8 (36 per cent) of the accidents attributable to maintenance and 13 (28 per cent) of those attributable to design or manufacture. The table is arranged in order of years in civil operation and a tendency can be detected for larger numbers of accidents to be attributed to older aircraft. This tendency is strengthened when fleet size is taken into account as shown in the brackets, but with a notable exception at the bottom of the table. Data on civil fleet size and years in operation were obtained from a separate source.

Analysis of the accident data by helicopter component is given in table 1 for 'potential airworthiness' causes. That is design, manufacture, and maintenance, plus those accidents in which the component was mentioned in the narrative, but the cause not positively identified. Maintenance causes are included under the term airworthiness because maintenance is an activity subject to the requirements of airworthiness authorities.

Engines head the list of component causes, at nearly 28 per cent followed by rotor systems with 27 per cent and transmissions at nearly 22 per cent.

Table 4 shows the accidents attributed to design or manufacture against each manufacturer by code, as a percentage of the number of aircraft sold (the figures for transmissions are shown in brackets), and from this a wide variation can be seen.

Table 3 shows the accident data by year (transmissions in brackets). Transmission related accidents attributed to design or manufacture are generally zero or one per year, with three exceptions: 1978 and 1983, when they reached three, and two in 1984.

Table 5 shows a breakdown of transmission related accidents by component for all causes. Tail rotor drive shaft heads the list with 32 per cent, followed by gears 19 per cent, and main rotor drive shaft 15 per cent.

It was not attempted in this analysis to determine the influence of rotor configuration because fleet size data was not available for one of the tandem rotor aircraft. Also, since much larger numbers of tandem rotor aircraft are used in military operations, it is recommended that analysis of military rotorcraft accidents be carried out to obtain guidance on this very important issue.

However, on another aspect of rotorcraft accidents - the likelihood of passenger survival - the data in the CAA Summary was analysed to determine whether rotor configuration might have been an influential factor. The criteria for this assessment was arbitarily set at fatalities exceeding 50 per cent of total compliment of passengers and crew, for which there were 63 accidents (17 per cent of the total). Of these 63 accidents tandem rotor aircraft were involved in four - that is 19 per cent of the total accidents involving tandem rotor aircraft compared with 17 per cent for the more conventional rotor configuration. The difference is not considered to be statistically significant even taking account of the disparity in configuration populations. The four accidents involving tandem rotor aircraft in which rotor synchronisation was lost resulted in total fatalities in three cases and 93 per cent in the third.

Of the 63 accidents referred to above, 62 per cent occurred during cruise flight (more than one minute after take-off), 21 per cent in logging, hoisting, or underslung load operations, 11 per cent in take-off or landing on helipads, buildings, ships decks, or oil rigs, 5 per cent in training or test flights, and 2 per cent in ground taxi at airports.
In Table 6, failure mode and cause are identified for each component, together with any history, and lessons are extracted for design, manufacture, maintenance of transmissions. In the final column, monitoring techniques are advocated which are considered to be appropriate to the problems identified. It would not be possible to assign confidence levels to these recommendations without more detailed knowledge of the failures.

Table 7, lists additional transmission-related accidents that are not included in the CAA World Helicopter Accident Summary by virtue of date, helicopter size, or military application. The accidents all involve fatigue fracture and the source is the Canadian National Aeronautical Establishment study on fatigue-related accidents. Whilst the report identifies the failed components it gives no information relating to the cause of the fatigue failure. This data is therefore not included in tables 1-6. This list is dominated by gear fractures and therefore strengthens the case for effective health monitoring for gear fracture, and for usage (torque) monitoring.

TABLE 1: ANALYSIS OF HELICOPTER ACCIDENT DATA BY COMPONENT

(Transport helicopters above 4550Kg maximum weight 1956-86)

Source: Analysis of data in CAA World Helicopter Accident Summary Iss. CA8, 1988

The table lists the number of accidents for which a particular component was identified in relation to deficiencies in design, manufacture, or maintenance, or cause unknown (but failed component identified) - i.e. potentially airworthiness-related.

Component	**Number of Accidents**	**Percentage of Total**
Engines	46	27.9
Rotors	45	27.3
Transmissions	36	21.8
Controls	16	9.7
Structure	13	7.9
Fuel	5	3.0
Electrical systems	4	2.4
TOTAL	165	100.0

TABLE 2: COMPARISON OF COMMERCIAL HELICOPTER ACCIDENT DATA (April 1956 - December 1986)

Source (Columns 4, 6-13): Analysis of data in CAA World Helicopter Accident Summary - Issue CA8, 1988 - Transport Helicopters greater than 4550Kg

Aircraft Type	Years in Oper-ation	No. Sold (Civil)	Total Accidents (% of fleet size in brackets)	CAUSE: Pilot, Grnd Crew, or Envi-ronment	CAUSE: Maintenance	CAUSE: Design or Manufacture	CAUSE: Not Known *
				(transmissions in brackets)			
A	32	172	103 (60)	43 (1)	6 (2)	9 (1)	45 (2)
B	30	N/A	4 -	2 (0)	0 (0)	1 (0)	1 (0)
C	29	16	13 (81)	6 (0)	2 (1)	2 (2)	3 (0)
D	25	139	59 (42)	24 (1)	4 (1)	14 (4)	17 (3)
E	16	20	9 (45)	1 (0)	1 (0)	0 (0)	7 (0)
F	16	511	84 (16)	46 (3)	4 (3)	5 (1)	29 (5)
G	16	125	24 (19)	9 (0)	2 (1)	0 (0)	13 (1)
H	12	85	29 (34)	10 (1)	1 (0)	5 (1)	13 (4)
I	9	265	26 (10)	8 (0)	2 (0)	5 (2)	11 (0)
J	7	34	1 (3)	0 (0)	0 (0)	1 (0)	0 (0)
K	6	92	2 (2)	0 (0)	0 (0)	1 (0)	1 (0)
L	6	85	3 (3)	1 (0)	0 (0)	0 (0)	2 (0)
M	5	11	4 (36)	1 (0)	0 (0)	3 (2)	0 (0)
TOTALS	-	1555	361 (23%)	151 (6)	22 (8)	46 (13)	142 (15)

* Failed components are frequently identified but not the root cause

TABLE 3: CIVIL HELICOPTER ACCIDENTS BY YEAR

(April 1956–December 1986)

(Transport helicopters greater than 4550Kg. Max. weight)

Source: Analysis of data in CAA World Helicopter Accident Summary, Iss. CA8, 1988

Transmissions: Those accidents involving transmissions are shown in brackets

Year	Total Accidents (All causes inc. Pilot)		Accidents Attributed to Design or Manufacture	
1956	2	(0)	0	(0)
1957	3	(0)	2	(0)
1958	5	(1)	0	(0)
1959	3	(0)	1	(0)
1960	1	(0)	1	(0)
1961	3	(0)	0	(0)
1962	0	(0)	0	(0)
1963	3	(1)	1	(0)
1964	6	(0)	0	(0)
1965	2	(1)	0	(0)
1966	9	(1)	1	(0)
1967	3	(0)	0	(0)
1968	11	(1)	1	(0)
1969	3	(0)	0	(0)
1970	4	(1)	2	(0)
1971	3	(1)	0	(0)
1972	8	(0)	1	(0)
1973	9	(0)	0	(0)
1974	14	(1)	1	(0)
1975	13	(3)	1	(0)
1976	19	(2)	0	(0)
1977	23	(2)	8*	(1)
1978	24	(7)	5	(3)
1979	13	(1)	1	(0)
1980	29	(5)	3	(1)
1981	27	(2)	3	(1)
1982	30	(4)	2	(1)
1983	28	(4)	7*	(3)
1984	17	(2)	3	(2)
1985	27	(0)	1	(0)
1986	19	(2)	1	(1)
TOTALS	361	(41)	46	(13)
% of total accidents	100	(11)	13	(4)

* includes accidents involving engine accessory gears (3 in total) – not included in brackets

TABLE 4: ANALYSIS OF CIVIL HELICOPTER ACCIDENTS ATTRIBUTED TO DESIGN/MANUFACTURE - BY MANUFACTURER

(Above 4550Kg max. weight, 1956-1986)

Source: Analysis of CAA World Helicopter Accident Summary, Iss. CA8, 1988

Manufacturer	Accidents attributed to design or manufacture as percentage of number sold (transmissions in brackets)	
A	18.5	(14.8)
B	5	(1.2)
C	1.8	(0)
D	1.5	(0.3)
E	0.5	(0)

TABLE 5: COMPONENT CONTRIBUTION TO TRANSMISSION - RELATED ACCIDENTS

(Source: Analysis of data in CAA World Helicopter Accident Summary)

COMPONENT	ACCIDENTS (all causes)*
Tail Rotor Drive Shaft	31.9%
Gears	19.1%
Main Rotor Drive Shaft	14.9%
Lubrication System	8.5%
Main Gearbox Input Shaft	8.5%
Bearings	4.3%
Freewheel	4.3%
Cooling Fan Drive	4.3%
Unknown	4.3%

* Accidents attributed to design, manufacture, maintenance, pilot/ground crew, or cause unknown - in which the failed component is identified.

TABLE 6 DETAILS OF ACCIDENTS INVOLVING TRANSMISSION FAILURE (All causes) - CAA Source

COMPONENT	FAILURE	CAUSE	HELICOPTER TYPES & HISTORY (*)	No of ACCIDENTS	LESSONS	MONITORING ADVOCATED
Tail Rotor Drive Shaft (TRDS)	Fracture	Overload, including cases of known propensity	3 *	4	DESIGN: Assume higher loads or provide means of limiting load. Make fixes mandatory. PILOT/GROUNDCREW: Better procedures for overload prevention required.	Tail torque, Underslung load, VHM
		Structure failure - contact with shaft	1	1	DESIGN/MANFR/MAINT: Raise criticality of structure in vicinity of drive shafts and rotors.	VHM
		Wear in universal joint - cause unknown	1	1	DESIGN/MANFR: More details of cause required. MAINT: Better control required.	VHM
		Incorrect part	1	1	MANFR/MAINT: Better control required.	VHM
		Unknown	3	5	Fit ADR's	VHM
	Spline coupling fracture	Inadequate lubrication	1 *	3	DESIGN: Avoid use of splined couplings where oil lubrication is impractical. MAINT: Better control required	VHM

TABLE 6 DETAILS OF ACCIDENTS INVOLVING TRANSMISSION FAILURE (All causes) - CAA Source

COMPONENT	FAILURE	CAUSE	HELICOPTER TYPES & HISTORY (*)	No of ACCIDENTS	LESSONS	MONITORING ADVOCATED
Main Rotor Drive Shaft (MRDS)	*Fracture*	*Lubrication problems*	*2*	*2*	*DESIGN*: Improve contamination control. MAINT: ditto	VHM/RTB
		Fatigue originating in rivet holes poorly drilled	1	2	MANFR: Better quality control required.	VHM/RTB
		Fatigue - no details	3	3	DESIGN/MANFR: Better quality control required.	VHM/RTB
MCB Input Shaft (MIS)	Fracture	Overload	1	1	As TRDS	Torque, USL, VHM
		Bearing failure	1 *	1	DESIGN: Ensure correct assessment of loads.	Brg. temp. VHM
		Lubrication failure (leak)	1	1	DESIGN: Provide on-line oil level (static) monitoring. MAINT: Better control reqd.	OLOLM
		Contact with structure	1	1	As TRDS	VHM
Cooling Fan Drive (CFD)	Fracture	FOD Damage to drive gear	1	1	MAINT: Better control reqd.	OLDM
		Double belt failure	1 *	1	DESIGN: Make fixes mandatory. MAINT: More diligence required with known faults.	Gear VHM and Brg. temp. to permit emerg. opn.

TABLE 6 DETAILS OF ACCIDENTS INVOLVING TRANSMISSION FAILURE (All causes) - CAA Source

COMPONENT	FAILURE	CAUSE	HELICOPTER TYPES & HISTORY (*)	No of ACCIDENTS	LESSONS	MONITORING ADVOCATED
Gears	Fracture - initiated at teeth	Misalignment	1 * (8 failures)	2	DESIGN: Analyse and make allowance for misalignment. Ensure fixes are mandatory and universally applied.	VHM, Torque, OLDM
		Unknown	2	4		OLDM, VHM, Torque
		Overload, 16 years since overhaul	1	1	MAINT } Better control reqd. PILOT }	VHM, Torque
	Fracture - initiated in bore.	1 locking failure - incorrect washers & assembly + 1 unconfirmed cause.	1	3	DESIGN: Locking and lubrication of splined gears require special attention. MANFR/MAINT: Better control required.	VHM, Torque
		1 from corrosion	1	1	DESIGN: Better environmental testing required	VHM
Bearings	R C fatigue-consequent gear fracture	Wear debris ignored	1	1	MAINT: Ensure monitoring provisions are understood and implemented	OLDM, VHM
		Unknown: Time in service more than 3000 hrs (TR pitch control bearing)	1 *	1	DESIGN: Ensure positive lubrication to such bearings.	Brg. temp.

TABLE 6 DETAILS OF ACCIDENTS INVOLVING TRANSMISSION FAILURE (All causes) - CAA Source

COMPONENT	FAILURE	CAUSE	HELICOPTER TYPES & HISTORY (*)	No of ACCIDENTS	LESSONS	MONITORING ADVOCATED
Lubrication System	Gearbox disintegration	Unknown for these 2 accidents, but history of pipe/connection failures and filter bowl failures	1 *	2	DESIGN: Avoid rigid external oil pipes and filter bowl distortion. Provide emergency lubrication system. MAINT: Check for leaks.	OLOLM + temperature of critical bearings
		Low oil	2	2	PILOT: Heed warning light	OLOLM
Clutch/ Freewheel	Failure in flight	Incorrect installation	1	1	MANUFACTURER/MAINT: Better control required.	
		Unknown	1(same)	1		
M G B Unknown			4	2	Fit ADR's	

LEGEND

- VHM: Vibration Health Monitoring
- RTB: Rotor Track and Balance
- OLDM: On-Line Debris Monitoring
- OLOLM: On-Line Oil Level Monitoring
- USL: Underslung load monitoring
- Brg temp: Bearing housing temperature

NB: Accidents listed comprise those attributed to transmissions, plus 3 accessory gearbox failures, and 2 rotor drive shaft failures

Source of data in columns 1-5: Analysis of data in CAA World Helicopter Accident Summary

TABLE 7 - HELICOPTER ACCIDENTS ATTRIBUTED TO FATIGUE FAILURE OF TRANSMISSION COMPONENTS - NAE SOURCE

KEY: Source: NAE = A Survey of Serious Aircraft Accidents involving Fatigue Fracture. Vol 2 NAE-AN-8, National Research Council, Canada. April 1983

Component: MGB = Main Rotor Gearbox; TGB = Tail Rotor Gearbox; IGB = Intermediate Gearbox; MDS = Main Rotor Driveshaft; TDS = Tail Rotor Driveshaft

	ACCIDENT	DATE	COMPONENT FAILED
1	BEARING FAILURES		
	1.1	6. 5.80	MGB No. 4 brg.
2	GEAR FAILURES		
	2.1	26.10.66	MGB Fan drive gears.
	2.2	28. 2.72	MGB Planet gear.
	2.3	2.10.73	MGB Drive shaft gear.
	2.4	18.10.74	Combining g'box - bevel gear.
	2.5	30. 5.77	Engine accessory drive inner bevel gear.
	2.6	30. 7.77	Engine accessory drive inner bevel gear
	2.7	1.12.77	MGB Ring gear drive coupling.
	2.8	14. 3.79	Combining gearbox - bevel gear.
	2.9	25. 7.80	MGB Output gear.
	2.10	4. 5.81	MGB Ring gear carrier
	2.11	24. 8.81	MGB Input pinion.
3	FREEWHEEL FAILURES		
	3.1	22. 4.70	MGB Freewheel
	3.2	2.10.77	Clutch assembly.

TABLE 7 - HELICOPTER ACCIDENTS ATTRIBUTED TO FATIGUE FAILURE OF TRANSMISSION COMPONENTS - NAE SOURCE

KEY: Source: NAE = *A Survey of Serious Aircraft Accidents involving Fatigue Fracture*. Vol 2 NAE-AN-8, National Research Council, Canada. April 1983

Component: MGB = Main Rotor Gearbox TGB = Tail Rotor Gearbox
IGB = *Intermediate Gearbox* MDS = Main Rotor Driveshaft
TDS = Tail Rotor Driveshaft

	ACCIDENT	DATE	COMPONENT FAILED
4	SPLINE/INTERNAL SHAFT		
	4.1	24. 8.79	Engine drive shaft.
	4.2	4. 4.80	MGB ring gear drive shaft.
5	LUBRICATION SYSTEM FAILURES		
	5.1	21.10.75	MGB Oil filter retaining bolt.
	5.2	2.12.76	" " " " "
6	GEARBOX CASING/ATTACHMENT FAILURE		
	6.1	31.10.80	Input flanges.
7	FAILURES WITH UNIDENTIFIED CAUSE		
	7.1	28. 2.73	MGB
	7.2	19.10.78	Engine gear assembly.
	7.3	10. 1.80	Engine gear assembly.

CHAPTER 2

DRIVE SYSTEM LIFE AND RELIABILITY

by

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INTRODUCTION

Fuel efficiency is an important objective in aircraft propulsion design. High speed gas turbine engines are light in weight. Propellers provide efficient aircraft propulsion. Yet, aerodynamics limits the speeds of fuel efficient propellers. Fan-jet or prop-fan engines are one way to obtain this desirable combination (Ref.1). Parallel axis geared drive systems are an essential part of the fan-jet engine. These drive systems take on many forms. However, they all contain gears and bearings. The gears and bearings transmit power from the high speed engine to the low speed propeller.

In design, the requirements of light weight and high reliability conflict. Designers use highly stressed, high quality alloy steels in the major load bearing components to resolve the conflict. This combination of high stress and high strength yields components which are light in weight. Unfortunately, they also have finite service lives. The lives are for millions of load cycles. However, at the high speeds of today's aircraft engines, millions of load cycles add up rapidly. This leads to frequent maintenance overhauls.

Lowering the cost of ownership is an objective in the design of today's more advanced aircraft (Ref.2). Fuel efficiency is one way to accomplish this. Yet, the gains in fuel costs can be lost in the hanger. The drive systems, which must be light in weight, must also be easy to maintain. As the components wear, they require maintenance. Good aircraft drive systems have long service lives between overhauls.

One estimate for the service needs of a drive system comes from its life and reliability models. The design objectives of long life between repairs and high reliability are worthy goals.

In this chapter, the statistics of drive system life and reliability are discussed. The statistics develop reliability models for repair prediction. The chapter discusses the modes of failure which are the basis for these models. Detailed coverage of the two-parameter Weibull distribution model for component and system life follows. Similar coverage of the Miner Palmgren load-life model also follows. Based on this model, the chapter presents the theory of mission spectrum averaging. Mission spectrum averaging determines the equivalent constant load which has the same life as the mission spectrum. The equivalent load extends the model from single load applications to the more common case of multiple power level applications.

The chapter then describes the component life and reliability models for bearings and gears. These drive system components require the most service. A treatment of the system life model follows. The chapter presents and compares two methods for estimating the drive system service schedule. One method uses a two-parameter Weibull life model for the entire drive system. The second uses exponentially distributed, approximate life models for the components.

Renewal theory is a supplemental statistical model. This theory describes the maintenance process. The theory considers the on going sequence of use, failure, repair and return to use. For this sequence, it predicts the number of replacement components needed to support the service maintenance schedule. A section covers this theory. The following section treats confidence theory. This theory applied confidence limits to repair frequency and replacement part supply estimates. The estimates provide a margin between the needs of a sample and the needs of the overall population.

The chapter then describes on-board monitoring systems. These monitoring systems actually call for the drive system overhauls. When a monitoring system detects the onset of a component fatigue failure, it alerts the pilot to schedule a repair. This continual checking greatly improves the reliability of the drive system and the aircraft.

Estimates of the drive system failure rate and replacement needs are still essential at the design stage. The estimates compare the relative worth of different designs. They help assess the cost of operating a proposed drive system design.

Numerical examples illustrate theory throughout the chapter. The intent of the chapter is to present the application of reliability theory to aircraft drive system design clearly.

RELIABILITY MODELS

Statistics can describe the life and reliability of the drive system components and the drive system itself (Refs. 3, 4, 5). There is a natural variation in the ability of each component to survive a given load. The load, which any given drive system component sees, also varies.

The reliability of a drive system is the probability that it will survive a given load. A drive system survives when it is able to continue operating at or above its design capacity. From the viewpoint of statistics, a drive system fails when it does not survive. In the analysis to follow, R describes the reliability of a drive system and F describes the probability of its failure.

Classical reliability theory includes no other possible result. The sum of the probabilities of survival and of failure is certainty. The equation for this is:

$$R + F = 1.0 \tag{1}$$

In the theory, there are no middle states. At some point in time or service, the drive system is no longer able to function at its design level. Before this point, it has survived. At and after this point, it has failed.

To make any failure benign, designers shift the point between survival and failure toward survival. Using the onset of failure as a failure event, service routines can maintain the drive system in a strong operating state. The probability of failure is really the probability of needing repair. A drive system which runs noisily, has debris in its oil, or has a lower efficiency, has this state of mathematical failure.

The life and reliability of a component are two independent functions of load. The load-life relationship is:

$$l_{10} F^p = K \tag{2}$$

l_{10} is the 90 percent reliability life of the component. F is the operating load. The power, p, is the load-life exponent. And K is a constant. Equation (2) states the load-life tradeoff at a fixed reliability. Increasing the expected component life reduces the load it can carry. Figure 1 is a log-log plot of the relationship with a load-life exponent of 3.5. This type of plot is the most common graphical description of fatigue strength versus life. Since component stress is proportional to component load, Equation (2) is the classical S—N fatigue relationship. It relates stress to cycles to failure for high strength steels.

The two-parameter Weibull distribution describes the statistical variations in reliability at a given load. Its equation is:

$$R = e^{-(l/\theta)^b} \tag{3}$$

R is the reliability of the component at the life, l. The parameter b is the Weibull slope. The parameter b describes the shape of the distribution. And the parameter θ is the characteristic life of the component. At this life, the reliability is 36.8 percent for any Weibull slope.

Figure 2 plots reliability versus life for a characteristic life of 100 million cycles and a Weibull slope is 1.5. Note the reduction in reliability as the number of service cycles increases. For the data of Figure 2, the l_{10} life at which the reliability is 90 percent is 22.3 million cycles. The l_{10} life is less than one-quarter of the characteristic life in this case.

Equations (2) an (3) relate the component's design life to its reliability for any load and service life.

MODES OF FAILURE

The components in a drive system can fail in many ways. If a drive system is under-designed, it can fail rapidly and completely in low cycle fatigue. An adequate design factor can prevent modes of failure such as housing fracture, buckling, gear tooth bending fatigue, and bearing race cracking. The use of a proper lubricant at an adequate operating temperature can prevent rapid failure also. Proper lubrication holds back modes of failure such as drive system overheating, bearing separator abrasion and gear tip scoring.

However, even in well designed, well lubricated drive systems, gradual component failures due to Hertzian pitting still happen. These failures occur in the compressive contact regions of the bearing races and the gear teeth faces. The failures produce pits and spalls in the load bearing area of the component surface.

Figures 3 through 5 are a series of photographs which show increasing Hertzian pitting damage in bearing race (Ref.6). The photographs show the final stages of this failure mode. The visible signs of failure follow a long process of invisible micro-slip damage. The micro-slip damage is hard to observe. It occurs at the grain or crystal level, at or near the surface of the metal. For bearings, this damage is internal. It occurs just below the surface of the bearing races. When the damage progresses enough it grows into a visible form.

In the first photograph of Figure 3, the bearing race contains a pattern of pits. These pits are the result of sub-surface shear stresses. Hertzian contact pressure produces the shear stresses. The shear stresses cause sub-surface cracks. The cracks then grow to the surface under repeated loading. On reaching the surface, the cracks remove small pits of material from the surface. These small pits start to occur only after many load cycles of fatigue damage.

In the second photograph of Figure 4, sections of the bearing race surface are missing. An extremely rough sub-surface is visible instead. The pieces of missing surface are spalls. They are flakes of material which have chipped out of the bearing race surface. The sub-surface cracks have grown together after many additional load cycles. Working together, these cracks make the spalls and the rough surface in the bearing race.

Figure 5 shows a structural crack in the bearing race. The stress concentrations from small sub-surface cracks can cause this type of failure. This crack is a rapid and complete failure of the bearing race. However, it only occurs at the end of a long chain of fatigue damage.

In generating the crack, the Hertzian stresses remove larger and larger pieces of material from the surface. These pieces become debris in the drive system oil. Magnetic plugs or other remote sensors in the drive system housing can detect their presence long before the final fracture. The presence of the surface pits also makes the bearings run noisily. Sound and vibration pickups can measure this property of a pitting failure as well.

Gradual pitting damage also occurs in gear teeth. Pitting damage identical to that in bearing races occurs at the pitch point on gear teeth faces. Pitting damage also occurs on the gear tooth face away from the pitch point in a slightly different way. Surface traction acts together with the compressive loading away from the pitch point. This is due to the relative sliding of the gear teeth. Figures 6 through 9 are a series of photographs of the sequence of Hertzian pitting damage on a gear tooth (Ref.7).

In the first photograph of Figure 6, the gear tooth has "gray stain" on its surface near the base of the tooth. This gray stain is a sign of micro-pitting at the surface of the gear tooth. These pits on tooth surface are smaller but are very similar to those for the bearing race. Although the micro-cracks begin at the surface, they produce surface pits just as the sub-surface cracks do for the bearing race. Figure 7 shows gear tooth pitting near the pitch point without gray stain. Sub-surface shear stresses cause these pits as they do for bearings. Figure 8 shows spalls produced by the growth of the tooth micro-cracks below the surface. Figure 9 shows a tooth fracture crack mid-way up the tooth face which the surface damage started.

As with bearing races, the Hertzian fatigue failure of a gear tooth begins with a long period of micro-crack initiation. The failure then causes surface pits which on-board failure monitoring systems can detect. After a period of pit growth, the failure starts producing spalls. If the drive system continues in operation, the failure can cause deep structural cracks. These cracks will produce tooth breakage which cause sudden and total drive system failure.

One can design the drive system to have a low level of contact pressure in these components. To resist surface damage further, other factors can be improved as well. Surface roughness, material strength and quality, oil film thickness and temperature all contribute to surface pitting failure. Aircraft designers and manufacturers use the best materials, processes and lubrication available to hold back surface pitting initiation. However, even well lubricated, high quality components, stressed at reasonable levels fail eventually in surface pitting.

It is important to design the drive system so failures will be gradual and not catastrophic. By predicting and detecting gradual failures, one can repair the drive system in maintenance sessions. It is also important to extend component lives without increasing aircraft weight. Any increase in component lives will increase the time between maintenance sessions. This will keep the aircraft drive system in service with a high reliability at a low service cost.

LIFE STATISTICS

The grain structure in a steel part is random. The start of surface pitting damage in a part is also random. Lives of bearings and gears in controlled test environments vary as the lives of components in service do. Since no design factor can remove the possibility of component failure, statistics is needed to describe component life.

The two-parameter Weibull distribution is the distribution most commonly used to describe fatigue life data. It can describe a wide variety of life patterns. The distribution also has the simple mathematical form of Equation (3).

In statistics, reliability is a double negative. Surviving is the state of not having failed. Statistics counts direct events. The act of failing is this type of event. A part can fail only once. It survives for its entire life. Thus the probability of failure is a direct statistic. The combination of Equations (1) and (3) describe it:

$$F = 1.0 - e^{-(l/\theta)^b} \tag{4}$$

F is the probability of failure expressed as a decimal. The constant e is the base of natural log. l is the component life in million load cycles or hours. θ is the characteristic life in million load cycles or hours. And b is the Weibull slope. This is the two-parameter Weibull probability distribution function for component failures. The two parameters are θ and b. The derivative of Equation (4) with respect to life is the probability density function, f:

$$f = \frac{b}{\theta}\left[\frac{l}{\theta}\right]^{b-1} e^{-(l/\theta)^b} \tag{5}$$

The probability density function is a histogram of life failures. Failures are single events. f, is the statistical function one normally sees. The function presents the scatter in the component lives.

Figure 10 shows the probability density function for a characteristic life of 100 million cycles and a Weibull slope of 1.5. The area under this curve is unity. This area represents the full set of failures for the component. The graph also shows the l_{10} life of 22.3 million cycles for the component. At the l_{10} life, only ten percent of the components have failed. Ninety percent continue to survive in service.

Figure 10 displays the median life, l_{50}, of 78.3 million cycles. The median life is the life at which one-half of all components have failed. This life divides the area under the probability density function into two equal halves.

The figure also shows the mean life, l_{av}, of 90.1 million cycles. The mean life is the numerical average of all component lives. Because the life distribution is not symmetric, the median and mean lives differ. For a Weibull slope of 1.5, the distribution has a skew to the high end of the distribution. This makes the mean greater than the median.

Figures 11 and 12 show the effect of Weibull slope on the shape of the distribution. Figure 11 is a plot of the probability density function for three distributions. All have a characteristic life of 100 million cycles. The three curves have Weibull slopes of 1.5, 3.5 and 5.5. The curve with a slope of 1.5 has a skew to the high end. The curve with a slope of 3.5 is nearly symmetric. This curve has a mean life nearly equal to its median life. And the curve with a slope of 5.5 has a slight skew to the low end. Its mean life is less than its median life. The Weibull slope is called the shape parameter because of this connection.

Figure 12 is a plot of the reliability function for the same three distributions. This figure illustrates the scaling property of the characteristic life. The characteristic life is the finite life at which the reliability is independent of the Weibull slope. All three reliability curves cross at the characteristic life. Its value defines the size of the distribution.

These two figures demonstrate that the Weibull slope also affects the spread of the distribution. Distributions with lower Weibull slopes have failures spread over larger ranges. Thus, the Weibull slope is also a spread parameter. However, the Weibull slope is not a spread parameter for the more general three-parameter Weibull distribution.

The three-parameter Weibull distribution includes a minimum life, θ_0, as the third parameter. Its equation is:

$$R = e^{-[(l-\theta_0)/(\theta-\theta_0)]^b} \tag{6}$$

In this form of the distribution, the Weibull slope, b, describes the shape of the distribution only. The two life parameters, θ and θ_0, describe the mean and spread of the distribution.

The two-parameter form of the distribution is by far more popular, due to its simplicity. One sizes the distribution to fit the data in the range of use. Often, the range of interest is from a reliability of one percent to a reliability of fifty percent. By fitting the failure data closely over this range, statistical component selection is possible. By using the simpler form of the distribution, the statistics are much easier to apply.

In working with the high reliability range, the l_{10} life often replaces the characteristic life as the scaling parameter. To eliminate θ, rearrange Equation (3):

$$\frac{1}{R} = e^{(l/\theta)^b} \tag{7}$$

Take logs of the expression:

$$\mathrm{Ln}\left[\frac{1}{R}\right] = \left[\frac{l}{\theta}\right]^b \tag{8}$$

Solve for characteristic life:

$$\theta = \frac{l}{\left[\mathrm{Ln}\left[\frac{1}{R}\right]\right]^{1/b}} \tag{9}$$

Now equate reliability, R, at any life, l, to 0.9 at l_{10}:

$$\frac{l}{\left[\mathrm{Ln}\left[\frac{1}{R}\right]\right]^{1/b}} = \frac{l_{10}}{\left[\mathrm{Ln}\left[\frac{1}{0.9}\right]\right]^{1/b}} \tag{10}$$

Solve for Ln(1/R):

$$\mathrm{Ln}\left[\frac{1}{R}\right] = \mathrm{Ln}\left[\frac{1}{0.9}\right]\cdot\left[\frac{l}{l_{10}}\right]^b \tag{11}$$

Although cumbersome, Equation (11) is the form that manufacturers use to present the two-parameter Weibull distribution characteristics of bearings (Ref.8). Using Equation (11) places 90 percent reliability lives in the catalogs.

In both Equations (8) and (11), the log of the reliability reciprocal is proportional to the life raised to the Weibull slope. Taking the log of either equation generates a straight line plot. Figure 13 is one such plot. The plot is a probability graph for the two-parameter Weibull distribution.

This graph helps one determine the distribution parameter values from fatigue test data (Ref.9). The plotted test data is the result of a series of identical life tests conducted on a sample set of identical components. The first failure determines the highest reliability data point. The next failure determines the next lowest reliability data point, and so on.

This data yields three lines which estimate the failure distribution. The solid straight line is the median estimate of the distribution. The two dashed curves are upper and lower confidence bounds for the distribution. The distribution for the larger set of all components can lie anywhere within these confidence limits.

Here lies one of the more confusing parts of statistics. Once one accepts all similar components as not exactly alike, one loses the certain knowledge of how any single component will behave. One also loses the certain knowledge of how any small group or subset of similar components will behave. Confidence statistics relate the behavior of a subset to the behavior of the entire group or universe of similar components.

In reliability determination, this double statistic appears twice. Quality control tests consume a subset to determine how all of the components will behave. Any given application uses a second subset of the components. Both test and use subgroups may behave differently from the universal set and from each other. The Weibull distribution estimates how any one component may behave within its subset. The confidence limits estimate how the subset's behavior may differ from the behavior of the entire group. The confidence limits remove certainty from the Weibull prediction.

Median ranking of the test failures gives the closest estimate of component reliability from the cycles to failure of the tested components. Five percent and ninety-five percent rankings of the test failures estimate the ninety percent confidence limits. There is a ninety percent probability that the total distribution lies somewhere within these limits. As the size of the test group increases, the spread between the confidence limits decreases. The confidence in the fit of the distribution increases as the spread decreases.

The closeness of reduced data to a fitted straight line in Figure 13 shows how well the distribution fits the data. If the data is linear, the two-parameter Weibull distribution describes the variations in component lives well. If it is not linear, the distribution does not. In this case, one should not use the two-parameter Weibull distribution. There are other distributions which may fit the data more closely. Fortunately, the Weibull distribution describes a wide variety of life distributions quite well.

From the plot of Figure 13, one can determine the Weibull parameters easily. The l_{10} life of 22.3 million cycles is on the graph at a reliability of 0.9. The characteristic life, $\theta = 100$ million cycles, is on the graph at a reliability of 0.368. To determine the Weibull slope, one can use two widely separated sets of life and reliability data. To obtain an expression for b, divide Equation 8 by itself for two different reliabilities:

$$\frac{\mathrm{Ln}(1/R_1)}{\mathrm{Ln}(1/R_2)} = \left[\frac{l_1}{l_2}\right]^b \tag{12}$$

Take logs and solve for b:

$$b = \frac{\mathrm{Ln}\left[\dfrac{\mathrm{Ln}(1/R_1)}{\mathrm{Ln}(1/R_2)}\right]}{\mathrm{Ln}\left[\dfrac{l_1}{l_2}\right]} \tag{13}$$

Equation 13 yields a direct calculation for the Weibull slope, b. In terms of l_{10} and θ, the value of b for the data of Figure 13 is:

$$b = \frac{\mathrm{Ln}\left[\dfrac{\mathrm{Ln}(1/0.9)}{\mathrm{Ln}(1/0.368)}\right]}{\mathrm{Ln}\left[\dfrac{22.3}{100}\right]} = 1.5 \tag{14}$$

With the plot of Figure 13 and the help of Equation (13), one can determine the Weibull parameters for the plotted data. These parameters are l_{10} = 22.3 million cycles and b = 1.5.

The two-parameter Weibull relationship describes the scatter in the component life about the l_{10} life at a give load. According to Equation (2), the l_{10} life varies inversely with the component load. In normal use, the two relationships are independent. So changes in the component load only change the l_{10} life. They do not change the Weibull slope or the validity of Equation (11).

LOAD-LIFE RELATIONSHIP

As with the Weibull relationship, the load-life relationship of common use is different from its definition form. The difference is the constant, K. One uses a load at a fixed life to elevate K. The dynamic capacity of the component, C, is the load which has a 90 percent reliability life of one million cycles. The load on the component is F. And the power, p, is the load-life exponent. In terms of the dynamic capacity, Equation (2) becomes:

$$l_{10} = \left[\frac{C}{F}\right]^p \tag{15}$$

The lives are in millions of load cycles. And the life at the dynamic capacity is one million load cycles. Thus, the dynamic capacity life does not appear as a variable in the equation. Figure 1 is a plot for a component with a dynamic capacity of 80 kN and a load-life exponent of 3.5.

Equation (15) states the load-life tradeoff at a fixed reliability. Increasing the expected component life reduces the load it can carry. Component stress is proportional to component load. Thus, the relationship is the classical S—N fatigue relationship of stress versus cycles to failure for high strength steels. It has no infinite life endurance strength. Equation (15) describes a continuing reduction in component capacity for increasing life over the entire range of possible design life. This relationship allows the various loads that a component sees to be combined into a single design equivalent load.

MISSION SPECTRUM AVERAGING

Figure 14 is a load-life curve for a component. The load-life exponent is p = 3.5. And the dynamic capacity is 80 kN. The graph shows three separate loads and their l_{10} lives. The loads are the major loads for a mission spectrum of component use. The Palmgren — Miner rule for averaging these loads is a linear cumulative damage rule. The result of the rule is an equivalent load, F_e. This load produces the same component fatigue damage as the mission spectrum does.

The rule states that any given load consumes a fraction of the fatigue life. The fraction is the ratio of the number of actual cycles to the number of allowed cycles at that load. The order of loading is not important. According to the rule, a part has certain fatigue life. The damage done at any load is that fraction of the allowable life consumed at that load. In terms of allowable l_{10} lives, this rule is:

$$\frac{l_e}{l_{10,e}} = \frac{l_1}{l_{10,1}} + \frac{l_2}{l_{10,2}} + \frac{l_3}{l_{10,3}} + \ldots \tag{16}$$

In Equation (16), $l_{10,e}$ is the 90 percent reliability life at the equivalent load, F_e. The 90 percent reliability lives, $l_{10,i}$, are the allowable lives which correspond to the applied loads, F_i. The lives, l_i, are the actual service lives at the applied loads, F_i. And the equivalent life, l_e, is the number of load cycles for F_e. This is the total number of load cycles:

$$l_e = l_1 + l_2 + l_3 + \ldots \tag{17}$$

To find F_e, substitute Equation (15) into Equation (16) for each l_{10} life. Note that the dynamic capacity and load-life exponent are the same for each term.

$$\left[\frac{F_e}{C}\right]^p l_e = \left[\frac{F_1}{C}\right]^p l_1 + \left[\frac{F_2}{C}\right]^p l_2 + \left[\frac{F_3}{C}\right]^p l_3 + \tag{18}$$

Substitute Equation (17) for l_e and solve for F_e:

$$F_e = \left[\frac{F_1^p l_1 + F_2^p l_2 + F_3^p l_3 + \ldots}{l_1 + l_2 + l_3 + \ldots}\right]^{1/p} \tag{19}$$

In Figure 14, the applied loads are: F_1 = 40 kN, F_2 = 25 kN, and F_3 = 18 kN. The l_{10} lives for these three loads are: $l_{10,1}$ = 11.3 million cycles, $l_{10,2}$ = 58.6 million cycles and $l_{10,3}$ = 185.1 million cycles. In the example, F_1 acts for 10 percent of the life, F_2 for 60 percent of the life, and F_3 for 30 percent of the life. So $l_1 = 0.1 \cdot l_e$, $l_2 = 0.6 \cdot l_e$ and $l_3 = 0.3 \cdot l_e$.

$$F_e = \left[\frac{40^{3.5} \cdot 0.1 + 25^{3.5} \cdot 0.6 + 18^{3.5} \cdot 0.3}{0.1 + 0.6 + 0.3}\right]^{0.2857} \tag{20}$$

$$F_e = 26.4 \text{ kN}.$$

The equivalent load for this mission spectrum is then F_e = 26.4 kN. From Figure 14, this load has a 90 percent reliability life of 48.3 million load cycles. This is the l_{10} life for the mission spectrum.

BEARING LIFE

The life model of this chapter is the model for rolling element bearings (Ref.8). Lundberg and Palmgren proposed the model in the late 1930s. They assumed that the log of the reliability of a bearing is proportional to its life, l, and some stress parameters. These parameters are: the stress level, τ_0; the depth to the maximum shear stress, z_0; and the stress volume, V. The relationship is:

$$\mathrm{Ln}\left[\frac{1}{R}\right] \sim \tau_0^c \, z_0^{-h} \, V \, l^b \tag{21}$$

In relation 21, b is the Weibull slope and c and h are exponents of proportionality. One must find the exponents c and h experimentally. This is the equation for the two-parameter Weibull distribution with the addition of stress and size factors. Relation 21 has led to Equations (11) and (15) for rolling element bearings.

The Anti-Friction Bearing Manufacturer Association (AFBMA) modified Equation (15) with adjustment factors. These factors extend Equation (15) to cover many different end use situations. As a result, rolling element bearing manufacturers and users size bearings with these relations. The revised code equation is:

$$l_{10,a} = a\left[\frac{C}{v\,F_r}\right]^p \tag{22}$$

In Equation (22), $l_{10,a}$ is the adjusted ninety percent reliability life for the bearing. The life adjustment factor is a. The load adjustment factor is v. And the equivalent radial load is F_r.

In Equation (22), the life adjustment factor is the product of a series of factors. These factors are: material quality, material processing, lubrication, speed and misalignment. The first three are improvement factors. They enable the designer to size bearings which are better than the 'standard' bearings in the catalogs. The improvements reflect advances in manufacturing and lubrication. The last two factors for speed and misalignment are de-rating factors. They allow the designer to compensate for application conditions which are not ideal.

The load adjustment factor determines an equivalent static load for comparison to the dynamic capacity. This is a de-rating factor. It includes factors for shock loading and race curvature correction. The purpose of the load adjustment factor is to estimate the actual maximum load from the nominal design load. The final term, F_r, in Equation (22) is the equivalent radial load for the bearing. This loa ' 'ombines the radial and axial load components into a single radial load which produces the same fatigue damage.

In Equation (11), the Weibull slope is 1.2 for ball and straight roller bearings. It is 1.5 for tapered roller bearings. In Equations (15) and (22), the load-life exponent may also have different values. For ball bearings, the value is 3.0. And for roller bearings, the value is 3.33.

Consider the design of a 373 kW planetary reduction. In the planetary reduction, a roller bearing supports each planet in the output arm. Figure 15 is a drawing of this application. The sun gear is the input. And the ring gear is the frame. The arm rotates at a constant speed of 345 rpm. The power level is steady. At this speed, the planets rotate at 1075 rpm relative to the arm. Each planet bearing carries a radial load of 29.26 kN. The load is stationary with respect to the arm. The bearing supports the planet in the arm with the inner race attached to the arm.

The selected roller bearing has a basic dynamic capacity of 108 kN. Its material is vaccum-induction melted, vacuum-arc remelted (VIM-VAR) AISI M-50 tool steel. The life adjustment factor includes a material and processing factor of 22 for the high quality steel selected. The lubrication factor is 0.3 This de-rating for planet bearing lubrication is due to low film thickness. The speed and misalignment factors are unity since no corrections are necessary. The combined life adjustment factor is a = 6.6. The load adjustment factor is v = 1.2. This factor is a de-rating for the rotation of the outer race relative to the load. The rotation places the fatigue damage on the inner race with its convex curvature. The outer race has lower Hertzian stresses on its concave curvature.

The design l_{10} life of this bearing is thus:

$$l_{10} = 6.6\left[\frac{108}{1.2 \cdot 29.26}\right]^{3.3} = 278 \tag{23}$$

The design l_{10} life of 278 million load cycles will provide 4316 hours of operation at the bearing speed of 1075 rpm.

Each application has its own speed and load environment. The code form of Equation (22) is helpful in extending laboratory component test data to fit real life situations. The basic reliability relationship of Weibull and fatigue relationship of Palmgren are thus available for product design.

GEAR LIFE

Gear teeth failures are similar to bearing failures. Due to the complexity of the gear tooth shape, there are some differences, however. Most of these differences take the form of sudden failures. The sudden failures occur in overloaded and poorly lubricated gear meshes.

As for rolling element bearings, a manufacturers association publishes design codes for gears (Ref.10). This association is the American Gear Manufacturers Association (AGMA). Its design codes deal primarily with the modes of sudden failure. Gear tooth fracture is the mode of failure which the AGMA codes treat most fully. The codes also treat scoring and pitting. The treatment of pitting is conservative at high stress levels but is not at lower stress levels.

Once again, the characteristics of slow pitting failure appear. Including the difference of possible surface initiation, gear pitting failures are very similar to bearing pitting failures. Surface sliding acts together with contact compression to complicate surface wear. However, abrasion is not part of the pitting failure. Abrasion can grind particles away from the surface, when the lubrication is inadequate. Abrasive failures begin immediately. They do not include a long period of invisible, pre-debris damage.

The physical cause of gear tooth pitting is the same as for bearing race pitting. Tested gear tooth life varies just as tested bearing life varies. So, engineers at the NASA Lewis Research Center formulated a model for gear tooth life similar to the bearing life model (Refs.7, 11).

The model builds upon the Lundberg-Palmgren theory. Starting with Equation (7.21), Coy, Townsend and Zaretsky developed a model for the reliability and life of a spur gear. The model uses both the two-parameter Weibull distribution of Equation (11) and the Palmgren load-life relation of Equation (15). With statistically replicated data, they showed that these models predict gear tooth pitting.

From Equation (21), they determined a relationship for the dynamic capacity, C_t, of a spur gear tooth. Rounding the exponents to one decimal, this expression is:

$$C_t = B\left[\frac{f}{\Sigma 1/p}\right] \cdot \left[\frac{1}{f L(\Sigma 1/p)}\right]^{1/10} \tag{24}$$

In Equation (24), B is a material strength which has the same units as stress. The effective face width of the tooth is f. The radial length of full load contact along the face of the tooth is L. And the curvature sum at the failure point for the contacting teeth is $\Sigma 1/p$. The curvature sum is:

$$\Sigma 1/p = \frac{1}{p_g} + \frac{1}{p_p} \tag{25}$$

Here, p_g is the radius of curvature of the gear tooth surface at the failure point. And P_p is the radius of curvature of the pinion tooth surface at the failure point.

The model of Equation (24) has two dimensionless groups. The first is a stress to strength ratio of $(C_t\ \Sigma 1/p)/(B\ f)$. The second is a size effect group raised to the 1/10 power. The size effect group brings stress volume and material grain size properties into the dynamic capacity model.

The dynamic capacity of the gear is lower than that of a tooth. Consider the simple case of a single pass, fixed axis gear set. Each rotation of the gear subjects each tooth on the gear to a single load cycle. The gear will fail when any single tooth on the gear fails. The fatigue damage in each tooth accumulates independently of the damage in the other teeth. In successive coin tosses, the probability of a specific combined event is the product of the probabilities of each coin toss. So too, the reliability of the gear, R_g, is the product of the reliabilities of each tooth in the gear.

$$R_g = R_t^{n_g} \tag{26}$$

In Equation (26), n_g is the number of teeth in the gear. And R_t is the reliability of a single tooth in the gear. The reliability of any tooth in a gear is equal to the reliability of any other tooth in the gear. To transform Equation (26) into a life relationship, substitute Equation (11) for the two reliabilities into the log of Equation (26).

$$\ell_{g10} = \frac{1}{n_g^{1/b}} \ell_{10} \tag{27}$$

The gear life, ℓ_{g10}, has units of million gear rotations. Unfortunately, Equation (27) is strictly valid for single mesh gears only. In this case, one gear rotation subjects each tooth to one load cycle. This is not the case for idler gears, collector gears and planetary gears. The different load count accumulation for these gears will change Equation (27) differently in each case. However the basic form of Equation (27) remains in each case. For idler gears, each tooth is loaded on two faces in a single rotation. Thus, one should multiply n_g by a factor, $d = 2$. Most other cases require a load count conversion factor, c. The factor, c, has units of tooth load cycles per gear rotation. The more general form of Equation (27) is:

$$\ell_{g10} = \frac{1}{c \cdot (d\, n_g)^{1/b}} \ell_{10} \tag{28}$$

Now substitute Equation (28) into Equation (15) for the tooth to obtain Equation (15) for the gear. This produces a gear dynamic capacity, C:

$$C = \frac{C_t}{c^{1/p} \cdot (d\, n_g)^{1/(b \cdot p)}} \tag{29}$$

The gears tested in the NASA experiments were vacuum-arc remelted (VAR) AISI 9310 steel gears. For these gears, the Weibull slope, b, is 2.5. The load-life exponent, p, is 4.3. And the material strength, B, is 120 MPa.

The initial fatigue data is for one set of test conditions. The tested gears were in a 1:1 reduction with 28 teeth on each gear. The module was 3.18 mm (8 diametral pitch). The contact face width was 2.8 mm. The test gear speed was 10,000 rpm. This produced a pitch line velocity of 47 meters per second. The test lubricant was a super-refined naphthionic mineral oil with a five percent extreme pressure additive package. The gear teeth had a 60-62 Rockwell C surface hardness and a 0.406 μm surface finish.

Consider the design of a 2:1 gear reduction to transmit 25 kW of power at a pinion speed of 2,000 rpm. The gears have a module of 2.12 mm (12 diametral pitch). Figure 16 shows the pinion which has 35 teeth in mesh with the gear which has 70 teeth. Both gears have a face width of 18.5 mm. The tooth proportions follow the AGMA standard for 20 degree pressure angle full depth teeth. The material is vacuum-arc remelted (VAR) AISI 9310 steel with a hardness of 60 Rockwell C. The teeth have a surface finish of 0.406 μm.

From gear geometry, the pinion has a pitch radius of 37.05 mm. Its teeth have a radius of surface curvature of 12.67 mm at the pitch point. The gear has a pitch radius of 74.1 mm and a radius of curvature of 25.34 mm. The length of contact in the full load region is 0.57 mm. For these values, the curvature sum is:

$$\Sigma 1/p = \frac{1}{25.34} + \frac{1}{12.67} = 0.118\ \text{mm}^{-1}. \tag{30}$$

The dynamic capacity of a single tooth is:

$$C_t = 120\left[\frac{18.5}{0.118}\right] \cdot \left[\frac{1}{18.5 \cdot 0.57 \cdot 0.118^2}\right]^{1/10} \tag{31}$$

$$C_t = 22.8\ \text{kN}.$$

The dynamic capacity of the pinion, C_p, is:

$$C_p = \frac{22.8}{(35)^{1/(2.5 \cdot 4.3)}} = 16.4\ \text{kN}. \tag{32}$$

The dynamic capacity of the gear, C_g, is:

$$C_g = \frac{22.8}{(70)^{1/(2.5 \cdot 4.3)}} = 15.4\ \text{kN}. \tag{33}$$

For this application, the normal load on the gear teeth is 3.43 kN. So the ℓ_{10} life of the pinion, ℓ_{p10}, is:

$$\ell_{p10} = \left[\frac{16.4}{3.43}\right]^{4.3} = 835 \text{ million cycles.} \tag{34}$$

At 2,000 rpm, a life of 835 million load cycles corresponds to a life of 6,950 hours. The ℓ_{10} life of the gear, ℓ_{g10}, is:

$$\ell_{g10} = \left[\frac{15.4}{3.43}\right]^{4.3} = 637 \text{ million cycles.} \tag{35}$$

At 1,000 rpm, a life of 637 million load cycles corresponds to a life of 10,620 hours.

As with the bearing life formula, one needs load and life adjustment factors to scale the test data to fit different applications. Factors such as lubrication, speed and surface finish will change the material strength of a gear. Additional test data and field service data correlation will determine these factors.

The model has value in its own right as an estimation of gear life and reliability. These predictions will complement those for bearing life and reliability. Similar life characteristics are now available from similar analytical tools.

The closeness of the models for bearing and gear life yields a further benefit. The models make a drive system life model possible. The system model combines the failure modes of different components into a single drive system model. With a system model, one can estimate a schedule to maintain the aircraft drive system in a high reliability state.

SYSTEM LIFE

The model for the life of a drive system is the strict series probabilitiy model (Ref.12). This model compares the system of load carrying gears and bearings to a chain of links. A chain fails when any single link breaks. So too, a drive system is in need of repair when any single component is in need of replacement or repair. In this model, the reliability of the system, R_s, is the product of the reliabilities of all the components.

$$R_s = \prod_{i=1}^{n} R_i \tag{36}$$

The high speed of drive system components and the spray of loose debris warrant the strict series probability model. If any component fails, debris may be present which could accelerate the fatigue damage in other components. So the drive system needs an overhaul to return it to a high state of reliability, when any element fails.

The log of the reciprocal of Equation (36) is:

$$\text{Log}\left[\frac{1}{R_s}\right] = \sum_{i=1}^{n} \text{Log}\left[\frac{1}{R_i}\right] \tag{37}$$

Substitution of Equation (11) into Equation (37) for each component yields:

$$\text{Log}\left[\frac{1}{R_s}\right] = \left[\frac{1}{0.9}\right] \sum_{i=1}^{n} \left[\frac{\ell_s}{\ell_{i10}}\right]^{b_i} \tag{38}$$

In Equation (38), ℓ_s is the life of the entire drive system for the system reliability, R_s. It is also the life of each component at the same drive system reliability, R_s. For consistency in Equation (38), all the component lives must have the same counting base. The unit chosen is millions of drive system output rotations.

Equation (38) is not a simple two-parameter Weibull relationship between system life and system reliability. The equation is a true two-parameter Weibull distribution only when all the Weibull exponents, b_i, are equal. In the general case, this is not true. However, a true two-parameter Weibull distribution can approximate Equation (38) quite well.

Equation (39) is the drive system two-parameter Weibull relationship. It includes the system reliability parameters, b_s and ℓ_{s10}.

$$\text{Log}\left[\frac{1}{R_s}\right] = \text{Log}\left[\frac{1}{0.9}\right] \cdot \left[\frac{\ell_s}{\ell_{s10}}\right]^{b_s} \tag{39}$$

The straight line reliability relationship of Equation (39) can be fit to the more exact relationship of Equation (38) numerically. The range used for this fit is $0.5 < R_s < 0.95$. Wider ranges may also be appropriate.

The method of fit is linear regression. The slope of the fitted straight line is the drive system Weibull slope, b_s. ℓ_{s10} is the life at which the drive system reliability, R_s, equals ninety percent on the straight line. This is the drive system ninety percent reliability life.

For an example, consider a simple drive system. Figure 17 shows a single mesh drive system. The system contains two gears and four support bearings. The pinion has 26 teeth and the gear has 39 teeth. The pinion turns at 3,000 rpm. The gear set transmits 100 kW power to an output gear torque of 480 N-m. The output speed is 2,000 rpm. The gears have a module of 4.23 mm (6 diametral pitch) and a face width of 25 mm.

For these conditions, the nominal tangential load on the gear teeth is 5.8 kN. The ℓ_{10} life of the pinion is 298 million output rotations, or 2,486 hours. The ℓ_{10} life of the gear is 380 million output rotations, or 3,170 hours. The Weibull slope for the gears is 2.5. The pinion has a dynamic capacity of 23 kN. The gear dynamic capacity is 24.6 kN. The load-life exponent for the gears is 4.3.

On both shafts, the bearings are 51 mm and 178 mm from their respective gears. Table 1 lists the loads on the four support bearings along with their lives and capacities. The gear and pinion loads, lives and capacities are in the table also. The Weibull slope for the bearings is 1.2. The load-life exponent for the bearings is 3.3.

Figure 18 is a Weibull plot for this drive system. It includes reliability lines for the pinion and the weakest bearing. The solid curve is for the component sum reliability as calculated from Equation (38). The dashed line is for the fitted system model of Equation (39). The fit of the system curve to the component sum reliability is normally this close or closer. In this example, the system Weibull slope is 1.57 and the drive system ℓ_{s10} life is 127 million output rotations or 1060 hours.

The analysis for the drive system dynamic capacity is similar. The basic dynamic capacity for the system is the drive system output torque D_s. This torque, D_s, produces a ninety percent reliability drive system life, ℓ_s, equal to one million output rotations. For these conditions, Equation (38) becomes:

$$1.0 = \sum_{i=1}^{n} \left[\frac{1}{\ell_{i10}}\right]^{b_i} \tag{40}$$

The output torque to component load ratio in a drive system is constant for all load levels. One can replace the actual and basic dynamic component loads with the corresponding drive system output torques in Equation (15) to obtain:

$$\ell_{i10} = \left[\frac{D_i}{T}\right]^{p_i} \tag{41}$$

In Equation (41), D_i is the component dynamic capacity in units of output torque. ℓ_{i10} is the component ninety percent reliability life. And T is the drive system output torque which produces the component ninety percent reliability life, ℓ_{i10}. For a ninety percent reliability drive system life of one million output rotations, T is the drive system dynamic capacity, D_s. Substituting Equation (40) for each component, gives:

$$1.0 = \sum_{i=1}^{n} \left[\frac{D_s}{D_i}\right]^{b_i p_i} \tag{42}$$

As with Equation (38), Equation (42) is not a simple load-life relationship. All component load-life exponents must be equal for the system load-life relationship of Equation (43) to result.

$$\ell_{s10} = \left[\frac{D_s}{T}\right]^{p_s} \tag{43}$$

One can solve Equation (42) numerically for D_s. First, iterate Equation (42) to determine a guess for the basic dynamic capacity of the drive system, D_{sg}. This value will provide a good range for the least square fit. The range of output torques for this fit is $0.1\ D_{sg} \leq T \leq D_{sg}$.

A log-log plot serves as the medium for the dynamic capacity versus life regression. The slope of the least squares fit is the negative of the reciprocal of the system load-life exponent, p_s. The drive system dynamic capacity, D_s, is on the regression line at the ℓ_{s10} life. D_s is the output torque at a ninety percent reliability life of one million output rotations for the drive system.

Figure 19 is the drive system load-life curve for the example of Figures 17 and 18. The fit of the straight line curve is better here than in Figure 18. In this example, the dynamic capacities of the pinion and gear are 1.8 and 1.9 kN-m respectively. The dynamic capacities of the four bearings are listed in Table 1. The drive system dynamic capacity is an output torque of 1.7 kN-m. The drive system load-life exponent is 3.74.

This model enables the designer to predict the service needs of a drive system in its design stage. Comparative evaluations of different designs are now possible on a reliability basis. Further correlations of predicted versus actual maintenance procedures will make the model even more valuable.

DRIVE SYSTEM SERVICE

The drive system service schedule should match the maintenance needs of the system. In the design stage, it is important to predict these maintenance needs accurately. Three models are available for this prediction. These are the Mean Time To Failure (MTTF), the Mean Time Between Repairs (MTBR) and renewal theory.

The Mean Time To Failure is the average failure time from the two-parameter Weibull distribution. It is the mean life of the component or system. The mean time is the first moment of the probability density function about zero time. Its calculation requires the use of the gamma function. The Mean Time To Failure is a direct statistic of the two-parameter Weibull distribution (Ref.5).

The Mean Time Between Repairs is a system calculation (Ref.5). It comes from the 'bathtub' curve in which the system life has three separate stages. The start-up period has a high infant mortality rate which decreases with time. The useful-life period has a constant, low mortality rate. And the wear-out period has an increasing mortality rate.

The exponential distribution describes the useful-life period with its constant mortality rate. This is a Weibull distribution with a slope of 1.0. Figure 20 shows the probability density function for an exponential distribution with a characteristic life of 100,000 hours. The mortality rate is the failure rate of the surviving population. The hazard function is the mortality rate. In contrast, the probability density function is the failure rate of the entire population.

Using the exponential distribution, the system failure rate is the sum of the component failure rates. For systems with a constant mortality rate and no replacements, this method calculates the mean time between failures or the Mean Time Between Repairs. Mean Time Between Failures and Mean Time Between Repair are other names for this method.

Renewal theory builds on the two-parameter Weibull life distribution for the component or system. It predicts the number of replacement parts required in the maintenance process. This theory recognizes that failures occur in a varying pattern.

All three estimates can use confidence theory to estimate the closeness of the application sample behaviour to the universal population behaviour.

MEAN TIME TO FAILURE

The Mean Time To Failure is the sum of all times to failure divided by the total number of failures. For a continuous distribution, the total number of failures is unity. The sum of all lives to failure is the integral of time or life multiplied by the probability density function. The range of the integral is from zero to infinity. The Mean Time To Failure is:

$$\mathrm{MTTF} = L_{av} = \int_0^\infty l\, f(l)\, dl \tag{44}$$

Substituting the probability density function of Equation (5) for the two-parameter Weibull distribution makes this:

$$\mathrm{MTTF} = \int_0^\infty l \frac{b}{\theta}\left[\frac{l}{\theta}\right]^{b-1} e^{-(l/\theta)^b}\, dl \tag{45}$$

To find the gama function in this integral, make the following substitution:

$$y = \left[\frac{l}{\theta}\right]^b \tag{46}$$

where

$$dy = \frac{b}{l}\left[\frac{l}{\theta}\right]^b dl \tag{47}$$

and

$$l = \theta\, y^{1/b}$$

Rearrange the integral:

$$\mathrm{MTTF} = \int_0^\infty l\left[e^{-(l/\theta)^b}\right]\frac{b}{l}\left[\frac{l}{\theta}\right]^b dl \tag{48}$$

and substitute Equations (46 and 47):

$$MTTF = \theta \int_0^\infty y^{1/b} e^{-y}\, dy \tag{49}$$

Since y is proportional to l in Equation (46), the integration limits of zero and infinity also apply for y. Equation (49) is the integral for the gamma function multiplied by the characteristic life, θ.

$$MTTF = l_{av} = \theta \cdot \Gamma(1 + 1/b) \tag{50}$$

For 1/b values between zero and one, a polynomial approximation for the gamma function is (Ref.13):

$$\Gamma(1 + 1/b) = 1 + \frac{a_1}{b} + \frac{a_2}{b^2} + \frac{a_3}{b^3} + \frac{a_4}{b^4} + \frac{a_5}{b^5} \tag{51}$$

The coefficients of this polynomial are:

$a_1 = -0.5748646$
$a_2 = +0.9512363$
$a_3 = -0.6998588$
$a_4 = +0.4245549$
$a_5 = -0.1010678$

The maximum error for the approximation is:

$$\text{Error} = \pm 5 \cdot 10^{-5}$$

If 1/b goes above one, a recursion formula will bring the gamma function back into this range. The formula is:

$$\Gamma\left(1 + \frac{1}{b}\right) = \frac{1}{b}\Gamma\left[1 + \left(\frac{1}{b} - 1\right)\right] \tag{52}$$

In terms of the l_{10} life, the MTTF is:

$$MTTF = \frac{l_{10} \cdot \Gamma(1 + 1/b)}{[\text{Ln}\,(1/0.9)]^{1/b}} \tag{53}$$

Figure 21 is a plot of the ratio of the two-parameter Weibull mean life to the characteristic life versus Weibull slope. The mean life equals the characteristic life at b = 1.0. It drops below the characteristic life to a minimum value at b = 2.15. It then increases back to the characteristic life as b approaches infinity. When b is infinite, the distribution is an impulse with all lives equal to the characteristic life.

Consider the system example of Figure 17. This example has an l_{10} life of 127 million output cycles or 1060 hours and a Weibull slope of $b_s = 1.57$. From Equation (53), the Mean Time To Failure is:

$$MTTF = \frac{1060 \cdot \Gamma(1.635)}{[\text{Ln}\,(1/0.9)]^{0.635}} = 3{,}974 \text{ hours.} \tag{54}$$

The drive system operating time of 3,974 hours corresponds to 477 million output rotations at a speed of 2,000 rpm. This is the estimated average time between required maintenances for the drive system based on the MTTF calculation. It is a direct function of the drive system's two-parameter Weibull distribution.

The Mean Time To Failure model assumes that each repaired drive system is as good as new. No distinction is made between the original drive system and a repaired drive system. This is not strictly true. However, one comparison of the MTTF model to field service data for an aircraft drive system showed remarkable agreement.

A joint group from NASA Lewis Research Center and the Allison Gas Turbine Division of General Motors performed a fatigue life analysis of the Allison T56/501 turbo-prop reduction gearbox (Ref.14). They plotted the times to failure for the gearbox field service data on two-parameter Weibull probability paper. The distribution fit the drive system failure data quite well. It had a Weibull slope of 1.3 and a mean life of 11,000 hours. For comparison, they performed a drive system analysis of the gearbox using the component life data and this system model. To raise the model life predictions up to the recorded field service lives, they had to use reasonable life adjustment factors. This comparison indicated that the model, on its own, is a conservative predictor of drive system life and repair frequency.

One can find the standard deviation of the two-parameter Weibull distribution by a similar gamma function integration.

The standard deviation is the square root of the second moment of the component life distribution about the mean. The second moment about the mean is the expectation of the square of the difference between the life and the average life.

$$\sigma_l = \left[\int_0^\infty (l - l_{av})^2 \cdot f(l)\, dl \right]^{1/2} \tag{55}$$

In terms of the gamma function, the standard deviation of the two-parameter Weibull distribution is:

$$\sigma_l = \theta \cdot [\Gamma(1 + 2/b) - \Gamma^2(1 + 1/b)]^{1/2} \tag{56}$$

The standard deviation of a distribution is a measure of the scatter of the distribution. The standard deviation will be valuable in estimating a confidence limit for the average life.

Figure 22 is a plot of the ratio of the standard deviation of the two-parameter Weibull distribution to its characteristic life. The figure plots the ratio versus the Weibull slope, b. At a slope of one, the distribution has a larger scatter. At this value, the distribution is the exponential distribution. As the slope increases to two, the scatter decreases rapidly. It then continues to decrease with the slope, b. At $b = \infty$, the distribution is an impulse and the standard deviation reaches zero.

MEAN TIME BETWEEN REPAIRS

The Mean Time Between Repairs or Mean Time Between Failures assumes a constant mortality rate for the system in service (Ref.5). This estimate is the most common reliability calculation in use. It models the lives of light bulbs, electronic components and many mechanical products which wear out and require replacement. The method depends on two strong assumptions, however.

The first assumption is that no replacement takes place. This is not really the case. Drive systems with failing components are repaired and returned to service. Replacement changes the counting base for the failure rate. The probability density function is the failure rate for the entire original population. The reliability is the fraction of the original population which is still in service. The hazard function is the mortality rate because it is the failure rate of the units still in service assuming no replacements. Thus, the hazard function is the probability density function divided by the reliability.

$$h = \frac{f}{R} \tag{57}$$

The hazard function for the two-parameter Weibull distribution is:

$$h = \frac{b}{l} \left[\frac{l}{\theta} \right]^b \tag{58}$$

Classic reliability theory divides the mortality rate into three distinct regions. These regions are: infant mortality, useful-life and wear-out. Figure 23 shows a typical 'bathtub curve' for a component or product which has these three regions of mortality rate. This curve is a plot of the hazard function versus product life or time.

In the infant mortality region, the mortality rate decreases with time. This is one reason manufacturers run many electronic products for a burn in period before shipment. The manufacturer does this to remove a high number of infant-mortality failures to assure a highy reliable product.

The second region of this curve is the flat, useful-life region. In this region, the mortality rate of the surviving products is constant. The mortality rate is also at an acceptably low level.

The third region of this curve is the wear-out region. In this region, the mortality rate increases with time. In its final stages, the rate becomes very large since the population base becomes very small. In this region, the product is past its useful life. If the onset of this region occurs after an acceptably long useful life, the product is good. One should replace any units reaching this age to maintain acceptable reliability. If the wear-out period begins too soon in a product's life, the product is of poor quality. Poor products should be redesigned to meet user expectations.

For products and components which are good, the useful-life region is the region of interest for scheduling maintenance. The second strong assumption of this method is that the mortality rate is constant. To have a constant mortality rate or hazard function, the two-parameter Weibull distribution must have a slope of $b = 1.0$. Manufacturers quality control tests show that it is not equal to one.

If b is one in Equation (58), the two-parameter Weibull distribution has a constant hazard function. This hazard function is equal to the reciprocal of the characteristic life. With b equal to one, the distribution simplifies to a one-parameter distribution called the exponential distribution. It is the distribution which serves as the basis for the Mean Time Between Repairs

calculation. The exponential distribution has a probability density function equal to:

$$f = \frac{1}{\theta} e^{-(l/\theta)} \tag{59}$$

And a reliability expressed by:

$$R = e^{-(l/\theta)} \tag{60}$$

The one parameter for the distribution is the characteristic life, θ. Equation (44) defines the mean life for a distribution. Substituting Equation (59) into Equation (44) yields:

$$l_{av} = -\int_0^\infty l[e^{-(l/\theta)}] \cdot \left[-\frac{1}{\theta} dl\right] \tag{61}$$

The method of parts will integrate this, with:

$$\begin{aligned} u &= l \qquad v = e^{-(l/\theta)} \\ du &= dl \quad dv = e^{-(l/\theta)} \cdot \left[-\frac{dl}{\theta}\right] \end{aligned} \tag{62}$$

By parts:

$$\int u\, dv = uv - \int v\, du \tag{63}$$

So:

$$l_{av} = -l e^{-(l/\theta)} \Big|_0^\infty - \theta \int_0^\infty e^{-(l/\theta)} \left[-\frac{dl}{\theta}\right] \tag{64}$$

This makes:

$$l_{av} = -l e^{-(l/\theta)} \Big|_0^\infty - \theta [e^{-(l/\theta)}]_0^\infty \tag{65}$$

Or:

$$l_{av} = -0 + 0 - \theta \cdot [0 - 1] = \theta = \text{MTBR} \tag{66}$$

So the mean life, l_{av}, of the exponential distribution is eq. .o the characteristic life, θ. This is the Mean Time Between Failures for the exponential distribution. Dividing Equation (59) by Equation (60) determines the mortality rate or hazard function for the exponential distribution.

$$h = \frac{1}{\theta} \tag{67}$$

The hazard function is the reciprocal of the Mean Time Between Repairs.

The standard deviation for the exponential distribution is:

$$\sigma_l = \left[\int_0^\infty (l - l_{av})^2 \cdot e^{-(l/\theta)} \cdot \frac{dl}{\theta}\right]^{1/2} \tag{68}$$

Integrating Equation (68) by parts twice, yields:

$$\sigma_l = l_{av} = \text{MTBR} \tag{69}$$

This large value for the standard deviation of the exponential distribution is a significant property of the distribution. The standard deviation, σ_l, is equal to the mean life, l_{av}. The distribution describes life data which have a high scatter.

Many systems have several different components. In this theory, each component can have its own constant failure rate. To determine the system failure rate, consider the strict series reliability of the system:

$$R_s = \prod_{i=1}^{n} R_i \tag{70}$$

The log of the reciprocal of Equation (70) is:

$$\mathrm{Ln}\left[\frac{1}{R_s}\right] = \sum_{i=1}^{n} \mathrm{Ln}\left[\frac{1}{R_i}\right] \tag{71}$$

Substitution of Equation (60 into 71) for all reliabilities yields:

$$\frac{1}{\theta_s} = \sum_{i=1}^{n} \frac{1}{\theta_i} \tag{72}$$

Thus the mortality rate for the system is the sum of the mortality rates for the components in the system.

$$h_s = \sum_{i=1}^{n} h_i \tag{73}$$

Now, match the two-parameter Weibull distribution to the exponential distribution for each component by equating the distribution means. The mean is a good measure of the central value of each distribution. By matching the means, one matches the distributions with the same expected value. This sets the mean life of the two-parameter Weibull distribution, MTTF, equal to the characteristic life of the exponential distribution, MTBR for the same component.

The Mean Time Between Repairs for the system thus becomes a direct function of the Mean Times To Failure for the components:

$$\mathrm{MTBR} = \frac{1}{\displaystyle\sum_{i=1}^{n} \frac{1}{\mathrm{MTTF}_i}} \tag{74}$$

For the system example of Figure 17, Table 1 lists the Mean Times To Failure for the components. Equation (53) relates the average life of a component to its l_{10} life and Weibull slope. Using these values, Equation (74) gives:

$$\mathrm{MTBR} = \frac{1}{\frac{1}{16{,}187} + \frac{1}{29{,}554} + \frac{1}{5{,}426} + \frac{1}{6{,}920} + \frac{1}{44{,}330} + \frac{1}{24{,}280}} \tag{75}$$

$$\mathrm{MTBR} = 2{,}050 \text{ hours.}$$

The Mean Time Between Repairs is also equal to 246 million output rotations. This compares to the values of 3974 hours and 477 million output rotations. These values are the maintenance period estimate of the last section. They are the Mean Time To Failure for the two-parameter Weibull system model.

The two predictions differ by a factor of two. The Mean Time Between Repairs estimate is for a life of one-half the Mean Time Failure estimate. Consider the assumptions made for both methods.

The Mean Time Between Repairs estimate assumes incorrect component failure distributions. It also assumes total independence of the component failure rates. The basis of the model is a level constant mortality rate of unrepairable components. For this condition, the system failure rate is the sum of the component failure rates. The model is valid for many complex electronic devices. However, this failure behavior is simply not correct for aircraft drive systems.

The Mean Time To Failure model uses the actual component failure distributions. It also assumes that the entire drive system is checked and restored in any maintenance session. The repair activity replaces all components which are near failure — not just the fully spalled component. The Mean Time To Failure calculation of the last section models the situation much more closely.

Neither method predicts the replacement part requirements directly in absolute time. The Mean Time Between Repairs method assumes no replacements. Although the component failure rate can serve as a crude replacement rate. The Mean Time To Failure method looks at the repairs in relative time. Its direct estimate of replacement needs would also be highly

conservative. A theory which does include the effects of repair and replacement for components in absolute time is renewal theory.

RENEWAL THEORY

In addition to scheduling maintenance periods, a service procedure must also estimate the numbers of replacement components required. Renewal theory adds the renewal function to the statistical tools for estimating repair. It estimates the number of replacements as a function of the component failure distribution and its life (Refs.15,16,17).

Renewal theory assumes the replacement of failed components as soon as they fail. This models an unending sequence of use and repair. Aircraft drive system maintenance follows this pattern closely. The renewal function results from a sequence of statistically predicted failures.

Consider the maintenance sequence. In a given life period, any number of failures may occur. The probability of at least one failure within a given life from the start of operation is:

$$F_1(l) = F(l) = \int_0^l f(x)\,dx \tag{76}$$

The probability of at least two sequential failures in the period is the probability of two independent events. The first component must fail. Then a second component must begin its service life at this failure life and also fail. The probability of having at least two failures in this period is:

$$F_2(l) = \int_0^l F_1(l - x) \cdot f(x)\,dx \tag{77}$$

In the integral of Equation (77), the life available for each failure changes as x increases from zero to l. The life for the first failure decreases. The life for the second failure increases. At $x = 0$, the entire life is available for the first failure probability. The probabilities of the second failure and the combination event are both zero. At the other extreme, x equals l. In this case, the entire life is available for the second failure probability. The probability of the first failure is zero. The probability of the combined event is thus zero as well at $x = l$. The integral defines a function for the probability of at least two failures in the life period from zero to l.

Equation (77) repeats indefinitely with increasing subscripts. The probability of having at least k failures in the period from zero to l is:

$$F_k(l) = \int_0^l F_{k-1}(l - x) \cdot f(x)\,dx \tag{78}$$

In Equation (78), $F_{k-1}(l - x)$ is the probability of having at least $k - 1$ failures in the period from zero to $l - x$.

Equations (76 through 78) can predict the probability of having exactly k sequential failures in the life period from zero to l. $F_{k+1}(l)$ is the probability of having at least k failures in the same period. These probabilities increase as the number of failures decrease. The more life available for a failure, the greater is the chance that it will occur. The probability of having exactly k failures is thus:

$$P_k(l) = F_k(l) - F_{k+1}(l) \tag{79}$$

This is the probability of having at least k failures less the probability of having at least k+1 failures.

The average number of failures in this life period is the expectation of the number of failures.

$$M(l) = \sum_{k=1}^{\infty} k\,P_k(l) = \sum_{k=1}^{\infty} k\,[F_k(l) - F_{k+1}(l)] \tag{80}$$

Since $(1 - 1)$ is zero, the last term of Equation (80) is:

$$\begin{aligned} -\sum_{k=1}^{\infty} k\,F_{k+1\,l} &= -\sum_{k=2}^{\infty} (k - 1)\,F_k(l) \\ -\sum_{k=1}^{\infty} k\,F_{k+1\,l} &= -\sum_{k=1}^{\infty} k\,F_k(l) + \sum_{k=1}^{\infty} F_k(l) \end{aligned} \tag{81}$$

This makes Equation (80) become:

$$M(l) = \sum_{i=1}^{\infty} F_i(l) \tag{82}$$

The mean number of failures is the infinite sum of the probabilities of at least i failures in the life period, *l*. This function, M*(l)*, is the renewal function. It is also expressed as:

$$M(l) = F_1(l) + \sum_{n=1}^{\infty} \int_0^l F_n(l - x) \cdot f(x)\,dx \tag{83}$$

and

$$M(l) = F_1(l) + \int_0^l M(l - x) \cdot f(x)\,dx \tag{84}$$

The derivative of the renewal function with respect to life is the renewal density function:

$$m(l) = f_1(l) + \int_0^l m(l - x) \cdot f(x)\,dx \tag{85}$$

These Equations give the number of replacements needed to support a maintenance schedule. Their solution involves a large series of convolution integrals. The Equations apply to any failure distribution. However, the solution is not easy to obtain. For a low-scatter failure distribution, the solution is an oscillation of replacement numbers about a straight line. The period of oscillation is close to the mean life. Tabulated solutions to the renewal function for the two-parameter Weibull distribution are available (Refs.16,18).

Figure 24 shows the renewal function for a component with a two-parameter Weibull life distribution. The component life has a Weibull slope of $b = 1.5$ and a characteristic life of $\theta = 5{,}000$ hours. For a Weibull slope of 1.5, the two-parameter Weibull distribution has enough scatter to minimize the renewal function oscillations. The large oscillations in the renewal function only appear for distributions with low scatter.

For distributions with scatter, an approximation for the renewal function is:

$$M_c(l) = \frac{l}{l_{av}} - \frac{l_{av}^2 - \sigma_f^2}{2 \cdot l_{av}^2} \tag{86}$$

The approximation accuracy increases as *l* increases. Equation (86) is an asymptote for the renewal means of low-scatter distributions. For high-scatter distributions, it approximates the renewal mean closely.

The renewal function is the probability of replacement for a single component. Its value goes above one because multiple replacements can occur. For a Q component system, the number of replacements is the product:

$$N_r = Q \cdot M(l) \tag{87}$$

Consider the case of an aircraft which uses ten similar bearings. Each bearing has a two-parameter Weibull life distribution with a characteristic life of 5,000 flight hours and a slope of 1.5. Equation (50) gives the Mean Time To Failure (MTTF) or average life of these bearings. It is:

$$\text{MTTF} = 5{,}000 \cdot \Gamma(1.666) = 4{,}515 \text{ flight hours.} \tag{88}$$

Equation (56) gives the standard deviation life for these bearings:

$$\sigma_f = 5{,}000\,[\Gamma(2.333) - \Gamma^2(1.666)]^{1/2} = 3{,}065 \text{ flight hours.} \tag{89}$$

Assume that a service facility must maintain fifty of these aircraft. Figure 24 and the table which it plots [Refs.16,18], give the renewal function values for these bearings. The number of components, Q, for use in Equation (87) is:

$$Q = 50 \cdot 10 = 500 \text{ bearings.} \tag{90}$$

For the first 4,000 flight hours, the facility will need:

$$N_{r1} = 500 \cdot 0.629 = 314 \text{ bearings,} \tag{91}$$

on the average, to maintain the aircraft. For the next 4,000 flight hours, it will need:

$$N_{r2} = 500 \cdot 0.873 = 437 \text{ bearings.} \tag{92}$$

The approximation formula of Equation (86) will give similar estimates:

$$\begin{aligned} M_c(4{,}000) &= \frac{4{,}000}{4{,}515} - \frac{4{,}515^2 - 3{,}065^2}{2 \cdot 4{,}515^2} = 0.617, \text{ and} \\ M_c(8{,}000) &= \frac{8{,}000}{4{,}515} - \frac{4{,}515^2 - 3{,}065^2}{2 \cdot 4{,}515^2} = 1.503. \end{aligned} \tag{93}$$

The difference is the renewal mean increase from the first to the second period:

$$M_c(8{,}000) - M_c(4{,}000) = 0.886. \tag{94}$$

Using these values in Equation (87), gives:

$$\begin{aligned} N_{r1} &= 500 \cdot 0.617 = 309 \text{ bearings,} \\ N_{r2} &= 500 \cdot 0.886 = 443 \text{ bearings.} \end{aligned} \tag{95}$$

Even for these low initial replacement times, the approximations are within six bearings of the more complete estimate for each period.

Figure 25 is a plot of the renewal function's standard deviation versus life for the two-parameter Weibull distribution which has a characteristic life of 5,000 hours and a slope of 1.5. This is the same distribution for the plot of Figure 24. The standard deviation for the two-parameter Weibull distribution's renewal function is tabulated with the renewal function mean (Refs.16,18).

The approximation for the standard deviation of the renewal function uses the third moment of the life distribution. For the two-parameter Weibull distribution, the third moment is:

$$\mu_3 = \int_0^\infty l^3 f(l)\, dl = \theta^3 \Gamma(1 + 3/b) \tag{96}$$

The approximation for the standard deviation of the renewal function is (Ref.16):

$$\sigma_{mc}(l) = \left[\frac{\sigma_l^2 \, i}{L_{av}^3} + \left[\frac{L_{av}^2 + \sigma_l^2}{4 L_{av}^4} \right] \cdot [3 L_{av}^2 + 5 \sigma_l^2] - \frac{2 \mu_3}{3 L_{av}^3} \right]^{1/2} \tag{97}$$

The standard deviation of the renewal function gives a measure of the scatter in replacement needs from one sample to the next. Estimates of replacement inventory need a margin for variations from the mean, as repair frequency estimates do. Confidence statistics provide one means for determining this margin.

CONFIDENCE STATISTICS

In predicting replacement rates and maintenance inventories, direct theory provides mean or average estimates. The Mean Time To Failure, Mean Time Between Repairs and mean replacements all come from the statistics of a universal population. With enough cases, these will be the true average values.

In any real situation, the number of drive systems under service is a limited sample. Confidence statistics estimate how differently a small sample may behave from its universal population. It uses the standard deviation of the universal failure distribution and the sample size to estimate the mean of the sample (Ref.18).

For many samples of the same size, the mean of the samples has a distribution about the overall mean. When the sample size is above thirty, the distribution is normal. The standard deviation of the means is:

$$\sigma_{av} = \frac{\sigma_l}{\sqrt{Q}} \tag{98}$$

In Equation (98), Q is the size of the sample. And σ_f is the standard deviation of the universal failure distribution.

In reliability predictions, the upper confidence bound has little significance. That a specific system is more reliable than the average is not a problem. Systems that are less reliable than the average are concerns, however. The normal distribution estimates the mean life which a greater percentage of all samples will have. This life is less than the mean life for the entire population. For a ninety percent confidence:

$$\ell_{av,90} = \ell_{av} - z_{10} \cdot \sigma_{av} \tag{99}$$

Here, z_{10} is the number of standard deviations below the mean which cuts off ten percent of the population. Ninety percent of the normal distribution lie above $\ell_{av,90}$. Ten percent lie below $\ell_{av,90}$.

Consider the effects of confidence on some estimates made earlier in the chapter. The system example of Equation (54) has an estimated average life of 3974 hours. This system has an ℓ_{s10} life of 1060 hours and a Weibull slope of $b_s = 1.57$. The characteristic life for the system is 4425 hours. From Equation (56), the standard deviation of the two-parameter Weibull distribution is:

$$\sigma_s = 4425 \cdot [\Gamma(2.270) - \Gamma^2(1.635)]^{1/2} = 2580 \text{ hours.} \tag{100}$$

If 100 drive systems require maintenance, Equation (98) estimates the standard deviation of the sample means as:

$$\sigma_{av} = \frac{2580}{\sqrt{100}} = 258 \text{ hours.} \tag{101}$$

For a 95 percent confidence, the mean time to failure for this sample of drive systems is:

$$\ell_{av,95} = \ell_{av} - z_5 \cdot \sigma_{av} \tag{102}$$

$$\ell_{av,95} = 3974 - 1.645 \cdot 258 = 3550 \text{ hours.} \tag{103}$$

There is a 95 percent confidence that the mean life of the sample is greater than 3550 hours for this drive system.

Consider the bearing inventory requirements of the example in the last section. The renewal function is a cumulative statistic. It predicts 314 replacements for 4,000 hours of operation and 751 total replacements for 8,000 hours of operation. The approximate model predicts 309 and 752 total replacements in the two periods.

A higher estimate of replacement bearings will cover variations in the sample from the total set of all bearings. Reversing the sign in Equation (99) provides this estimate:

$$N_{av,90} = N_{av} + z_{10}\, \sigma_{av} \tag{104}$$

For a ninety percent confidence of having enough bearings, one needs the standard deviations in the renewal function. From Figure 25, these are:

$$\begin{aligned} \sigma_{m1} &= 0.704 \text{ at 4,000 hours, and} \\ \sigma_{m2} &= 0.973 \text{ at 8,000 hours.} \end{aligned} \tag{105}$$

Apply Equation (98) to obtain the standard deviations of the means. This divides both standard deviations by $Q^{1/2}$. Then multiply by Q to get the replacement standard deviation, σ_{av}.

$$\sigma_{av} = Q \frac{\sigma_m}{\sqrt{Q}} = \sqrt{Q}\, \sigma_m \tag{106}$$

The two replacement standard deviations are:

$$\begin{aligned} \sigma_{av1} &= 500^{1/2} \cdot 0.704 = 16 \text{ bearings at 4,000 hours, and} \\ \sigma_{av2} &= 500^{1/2} \cdot 0.973 = 22 \text{ bearings at 8,000 hours.} \end{aligned} \tag{107}$$

Applying Equation (104) twice gives:

$$\begin{aligned} N_{av1,90} &= 314 + 1.282 \cdot 16 = 334 \text{ bearings at 4,000 hours, and} \\ N_{av2,90} &= 751 + 1.282 \cdot 22 = 779 \text{ bearings at 8,000 hours.} \end{aligned} \tag{108}$$

The difference is 445 additional bearings for the second period. These twenty-eight additional bearings increase the chance of having an adequate repair stock from fifty percent to ninety percent.

To use the approximation model of Equations (86 and 97) for these calculations, one needs the third moment of the distribution. From Equation (96):

$$\mu_3 = 5{,}000^3 \cdot \Gamma(3) = 250 \cdot 10^9 \text{ hours}^3. \tag{109}$$

Use equation (97) to find the renewal standard deviations:

$$\sigma_{m1} = \left[\frac{3{,}065^2 \cdot 4{,}000}{4{,}515^3} + \left[\frac{4{,}515^2 + 3{,}065^2}{4 \cdot 4{,}515^4}\right] \cdot [3 \cdot 4{,}515^2 + 5 \cdot 3{,}065^2] - \frac{2 \cdot 250 \cdot 10^9}{3 \cdot 4{,}515^3}\right]^{1/2} = 0.731, \text{and}$$

$$\sigma_{m2} = \left[\frac{3{,}065^2 \cdot 8{,}000}{4{,}515^3} + \left[\frac{4{,}515^2 + 3{,}065^2}{4 \cdot 4{,}515^4}\right] \cdot [3 \cdot 4{,}515^2 + 5 \cdot 3{,}065^2] - \frac{2 \cdot 250 \cdot 10^9}{3 \cdot 4{,}515^3}\right]^{1/2} = 0.971 \tag{110}$$

Once again, these values are very close to the renewal function values from the chart (Ref.16). Now use Equation (106) to find the replacement standard deviations.

$$\begin{aligned} \sigma_{av1} &= 500^{1/2} \cdot 0.731 = 16 \text{ bearings at 4,000 hours, and} \\ \sigma_{av2} &= 500^{1/2} \cdot 0.971 = 22 \text{ bearings at 8,000 hours.} \end{aligned} \tag{112}$$

The approximation estimated the average number of repairs at 309 for 4,000 hours and at 752 for 8,000 hours. Applying Equation (104) twice gives the ninety percent confidence estimates:

$$\begin{aligned} N_{av1.90} &= 309 + 1.282 \cdot 16 = 330 \text{ bearings at 4,000 hours, and} \\ N_{av2.90} &= 752 + 1.282 \cdot 22 = 780 \text{ bearings at 8,000 hours.} \end{aligned} \tag{113}$$

The difference is 450 additional bearings for the second period. Thus, twenty-eight additional bearings will increase the confidence from fifty to ninety percent.

One should note the closeness of this approximation to the more exact and more involved convolution solution. The differences between the two estimates are well within the statistical accuracy of the entire process. No significant loss occurs in renewal theory estimates with Equations (89) and (97). And rapid estimates are possible. These estimates include confidence levels as well.

Since the behavior of samples differs from the behavior of the 'ideal' distribution, confidence estimates are helpful. With the confidence estimate, one can see the effects of sample size on the life and replacement estimates. These estimates help in the design stage of a drive system's development.

However, actual lives and replacement numbers will differ from these estimates. The estimates are normally conservative. Sometimes they are extremely conservative. In aircraft use, a more reliable indication of the need for maintenance is on board monitoring systems.

SUMMARY

Aircraft drive systems must be light in weight. They also must transmit high power at high speeds. This combination of requirements make finite life design a reality. Statistics are essential to properly evaluate drive system designs. The mean life and life scatter should be understood and controlled at the design stage of a drive system's development. This chapter has treated the statistics of drive system life and reliability.

The fundamental statistic for component life is the two-parameter Weibull distribution. This distribution is dominant in rolling element bearing life description. It also describes other fatigue life data well. The two parameters of the distribution are the characteristic life and the Weibull slope. The characteristic life scales the distribution. The Weibull slope shapes it.

The chapter described and illustrated experimental techniques for obtaining the two parameter Weibull distribution which best fits a component life pattern. Sample size effects on the confidence level of the distribution fit were discussed also. Confidence is a secondary statistic. It describes the probability that any specific sample of components has the same life pattern as the total component population.

The Weibull distribution frequently combines with a linear relationship of log load versus log life. The distribution and the relationship describe component capacity and life. The chapter described the distribution, the load-life relationship and the modes of failure which produce these failure characteristics. The use of the load-life relationship to convert a mission spectrum of loads into a single equivalent design load was described.

The specifics of component life models were then presented. Relationships between fundamental material properties and the geometry and loading of the components were treated. Bearing life models and gear life models were treated separately. The effects of multiple failure locations on gear life were considered. The gear dynamic capacity was determined from the tooth capacity using basic reliability theory.

These component life and capacity models were then combined into drive system life and capacity models. The system life model is also a two-parameter Weibull distribution. Basic reliability theory was once again important in the development of these models. Examples were given to illustrate the method.

The application of the models to the estimation of drive system service requirements was then treated. The Mean Time To Failure method used the drive system two-parameter Weibull life distribution in relative time. The Mean Time To Failure model provided direct estimates of failure rates and repair frequency. The Mean Time Between Repair method provided other estimates of the failure rates and repair frequency. The MTBR method and its use of the exponential distribution was explained in the chapter. A system example was presented to compare the two methods quantitatively. The superiority of the two-parameter Weibull drive system life model was shown. The two-parameter Weibull Mean Time To Failure method models the repair frequency for aircraft drive systems more closely than the Mean Time Between Repairs method.

Renewal theory was then presented to complement the repair frequency estimates with a spare parts supply need estimate. To repair aircraft drive systems quickly, an adequate supply of spare components must be on hand. Single sided confidence statistics were then developed. The confidence limits were applied to the estimates of repair frequency and spare parts supply. Confidence statistics give estimates of repair frequency and supply quantity which are conservative relative to the mean estimates. These conservative estimates improve the confidence that the repair frequency and part supply will be adequate for an actual sample. A small sample normally behaves differently from the universe of drive systems described by the direct statistics.

Drive systems require design to minimize the overall maintenance requirements. In order to reduce the cost of ownership, the drive system must be reliable. The methods of this chapter can serve as estimators for the reliability of competing designs before they are built. The objective is to bring only the best designs to the stages of testing and adoption.

NOMENCLATURE

a	bearing life adjustment factor
b	Weibull slope
B	material strength constant (MPa)
c	load count factor
C	dynamic capacity (N)
d	tooth load factor
D	dynamic capacity
e	base of the natural log
f	probability density function
f	face width (m)
F	load (N)
F	probability distribution function- probability of failure
F_k	probability of at least k failures
h	hazard function — mortality rate
l	life (million cycles or hours)
L	length of high stress region (m)
Ln	natural log
M	renewal function
M_e	approximate renewal function
MTBR	Mean Time Between Repairs for system
MTTF	Mean Time To Failure — average life
n	number of teeth
N_r	number of replacements
P_k	probability of exactly k failures
Q	sample size
R	probability of survival — reliability
T	output torque (N-m)
v	bearing load factor
V	stress volume (m^3)
x	dummy time variable (hours)
z_o	depth to maximum shear stress (m)
z_5	number of standard deviations from the mean which cuts off a 5 percent population tail
z_{10}	number of standard deviations from the mean which cuts off a 10 percent population tail

Γ	the gamma function
μ_3	third moment of a probability density function
ρ	radius of curvature (m)
σ	standard deviation
θ	characteristic life (million cycles or hours)
θ_o	minimum life (million cycles or hours)
τ_o	maximum shear stress (MPa)

exponents

b	Weibull slope
c	dimensional exponent
h	dimensional exponent
n	total number
p	load-life factor

subscripts

a	adjusted
av	average or mean
e	equivalent single value
f	initial distribution
g	gear
i	index
k	index
m	renewal function
me	approximate renewal function
p	pinion
r	equivalent radial value
s	system
t	tooth
1,2,3	index values
10	ten percent failure — ninety percent reliability
50	fifty percent reliability — median

REFERENCES

1. Ludemann, S.G. "Prop-Fan Powered Aircraft — An Overview"; SAE Paper 820957, August 1982.
2. Ford, D. "Reducing the Total Cost of Ownership — A Challenge to Helicopter Propulsion System Specialists"; American Helicopter Society Rotary Wing Propulsion System Specialists Meeting Paper RWP — 6; Williamsburg, Va.; November 1986.
3. Barlow, R.E., Proschan, F. *Statistical Theory of Realibility and Life Testing;* Holt, Rinehart and Winston, Inc.; New York; 1975.
4. Kapur, K.C., Lamberson, L.R. *Reliability in Engineering Design;* John Wiley & Sons, Inc.; New York; 1977.
5. Goldberg, H. *Extending the Limits of Reliability Theory;* John Wiley & Sons, Inc.; New York; 1981.
6. Parker, R.J. "Large-Bore Tapered-Roller Bearing Performance and Endurance to 2.4 Million DN"; *Advanced Power Transmission Technology;* NASA CP2210; 1982; pp.253—269.
7. Coy, J.J., Townsend, D.P., Zaretsky, E.V. "An Update on the Life Analysis of Spur Gears"; *Advanced Power Transmission Technology;* NASA CP2210; 1982; pp.421—433.
8. Harris, T.A. *Rolling Bearing Analysis;* 2nd Edition; John Wiley & Sons, Inc.; New York; 1984.
9. Johnson, L.D. *The Statistical Treatment of Fatigue Experiments;* Elsevier Scientific Publishing Co.; Amsterdam; 1974.

10. — "AGMA Standard for Rating the Pitting Resistance and Bending Strength of Spur and Helical Involute Gear Teeth"; American Gear Manufacturers' Association Standard 218.01; Arlington, Va.; December 1982.

11. Townsend, D.P; Coy, J.J.; Zaretsky, E.V. "Experimental and Analytical Load-Life Relation for AISI 9310 Steel Spur Gears"; *ASME Journal of Mechanical Design;* Vol. 100; No. 1; January 1978; pp.54—60.

12. Savage, M.; Radil, K.C.; Lewicki, D.G.; Coy, J.J. "Computerized Life and Reliability Modeling for Turbo-Prop Transmissions"; NASA TM 100918, 1988.

13. Abramowitz, M.; Stegun, I.A. *Handbook of Mathematical Functions with Formulas, Graphs & Mathematical Tables;* National Bureau of Standards Applied Mathematics Series * 55; Washington, D.C.; June 1964.

14. Lewicki, D.G.; Black, J.D.; Savage, M.; Coy, J.J. "Fatigue Life Analysis of a Turbo-Prop Reduction Gearbox"; ASME Journal of Mechanisms, Transmissions, and Automation in Design; Vol. 108; June 1986; pp.255—262.

15. Cox, D.R Renewal Theory; Chapman & Hall; New York; 1982.

16. White, J.S. "Weibull Renewal Analysis"; Third Annual Aerospace Reliability & Maintainability Conference Proceedings; Washington, D.C.; June 1964; pp.639—657.

17. McCall, J.C. "Renewal Theory — Predicting Product Failure and Replacement"; Machine Design; Vol. 48; No. 7; March 1976; pp.149—154.

18. Lipson, C.; Sheth, N.J. Statistical Design and Analysis of Engineering Experiments; McGraw Hill Book Co.; New York; 1973.

Table 1
Single Mesh Drive System Properties

	Load N	l_{10} 10^6 Cyc.	l_{10} Hours	l_{av} Hours	Dyn. Cap. kN	Dyn. Cap. kN-m
Bearing 1	4.1	317	2,640	16,187	23.6	2.75
Bearing 2	10.3	578	4,820	29,554	70.8	3.3
Pinion	5.8	298	2,486	5,426	23.0	1.8
Gear	5.8	380	3,170	6,920	24.6	1.9
Bearing 3	10.3	868	7,230	44,330	80.0	3.73
Bearing 4	4.1	475	3,960	24,280	26.7	3.1

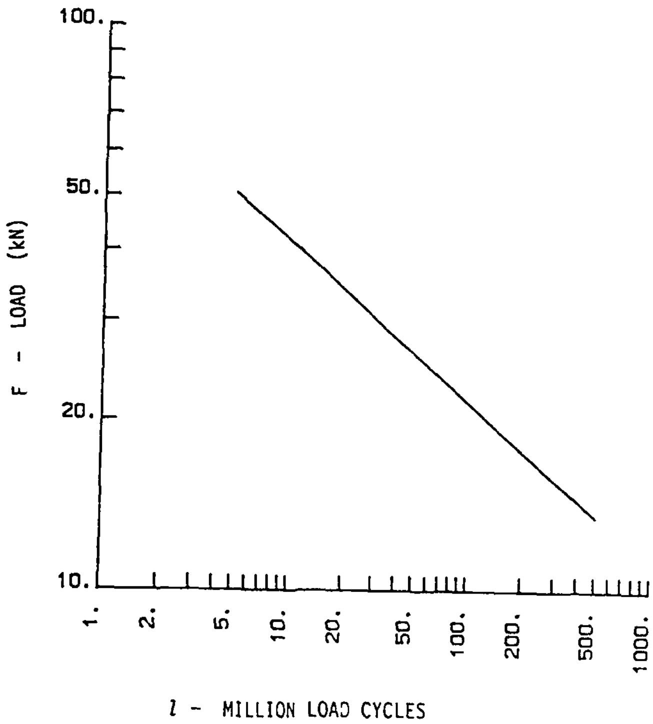

Fig.1 Load-life curve

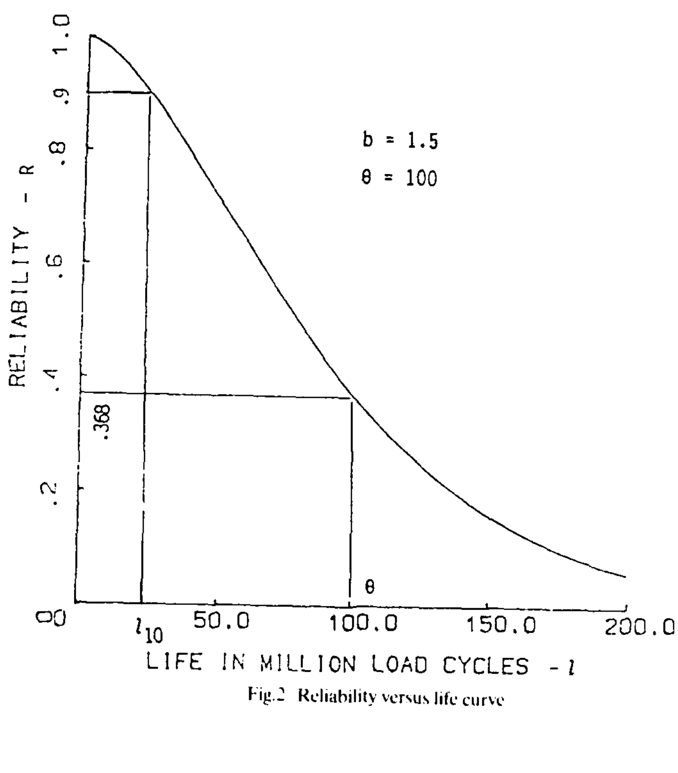

Fig.2 Reliability versus life curve

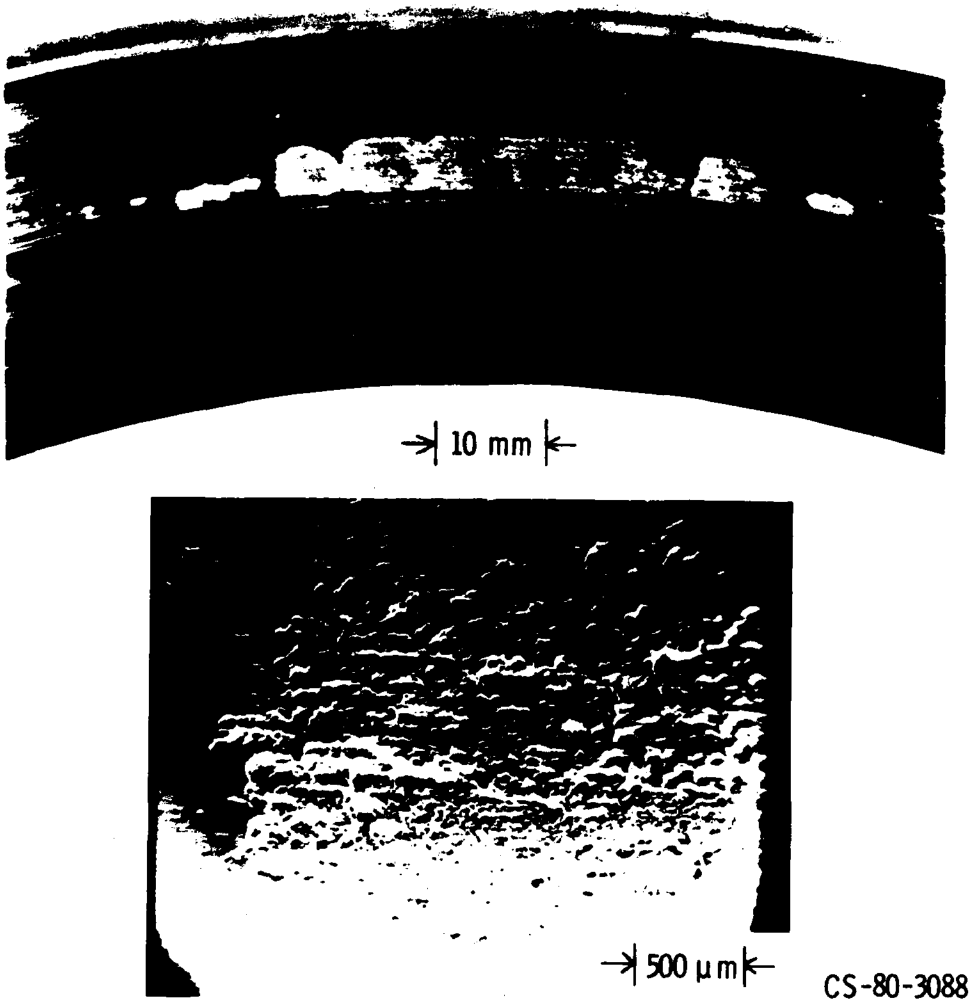

Fig.3 Peeling failure on cup raceway surface after 569 hours with standard design bearing run at 12,500 rpm

Fig.4 Hertzian bearing spalling on cone raceway

Fig.5 Bearing cone race crack

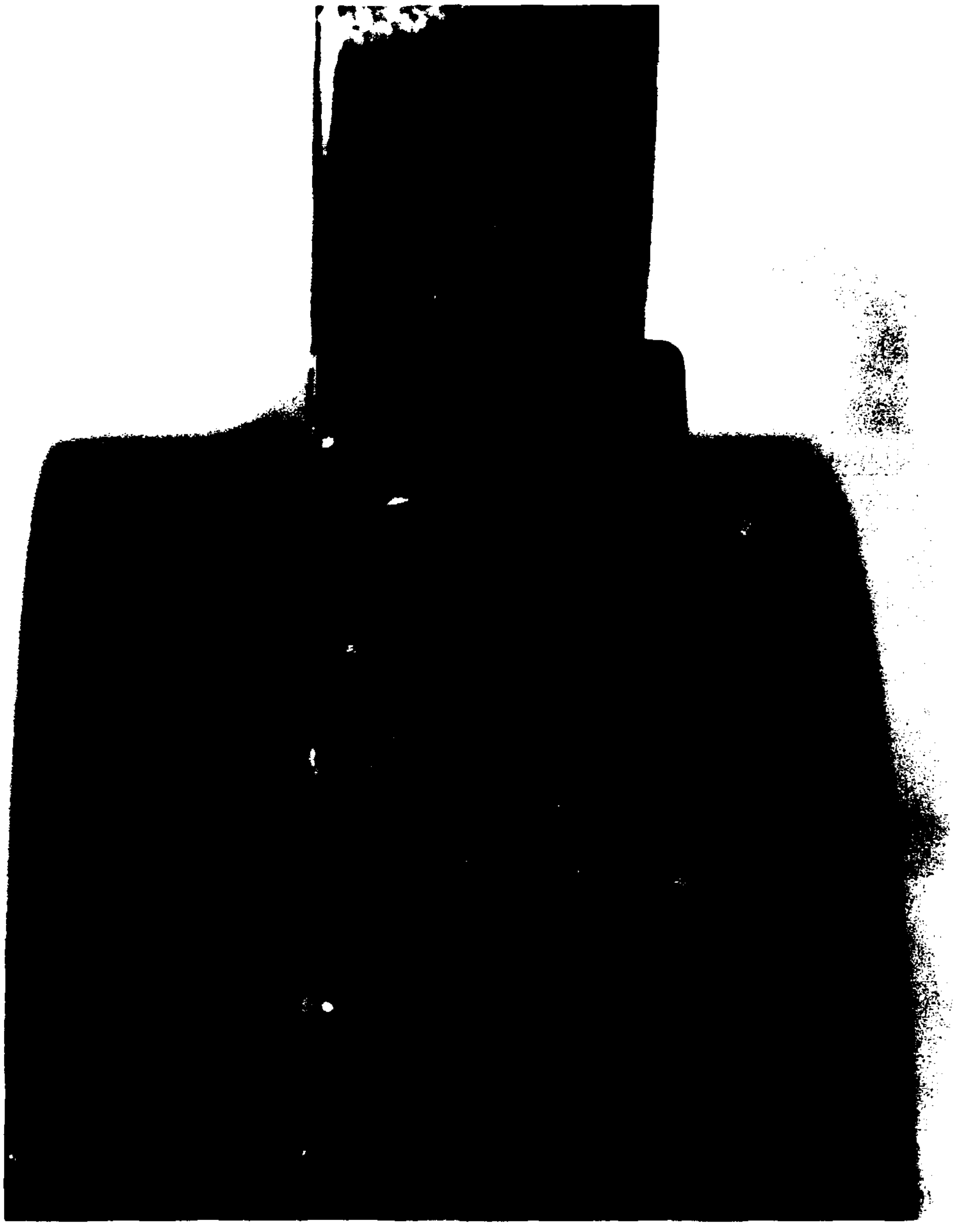

Fig.6 Gear tooth gray stain

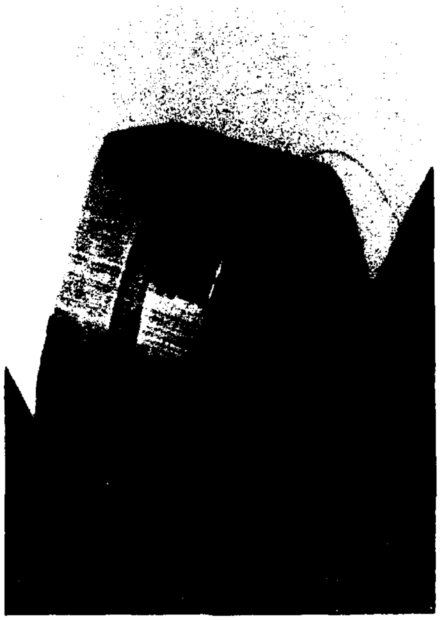

Fig.7 Gear tooth pitting

Fig.8 Gear tooth spalls

Fig.9 Gear tooth crack

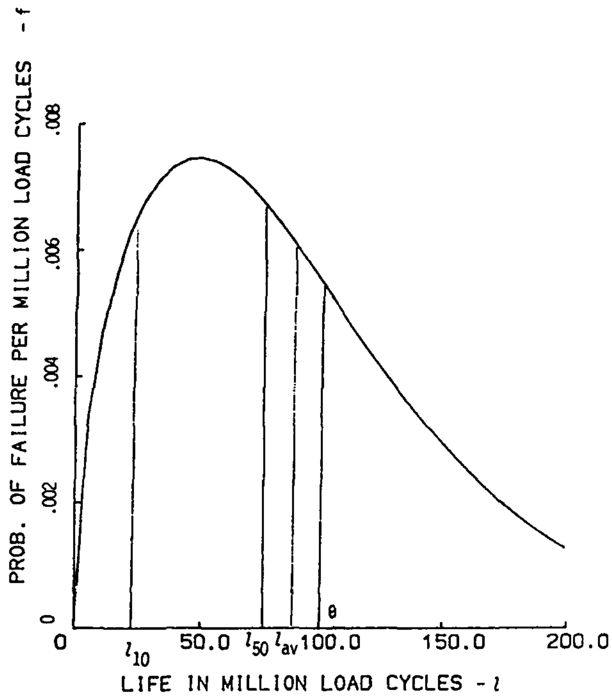

Fig.10 Life probability density function

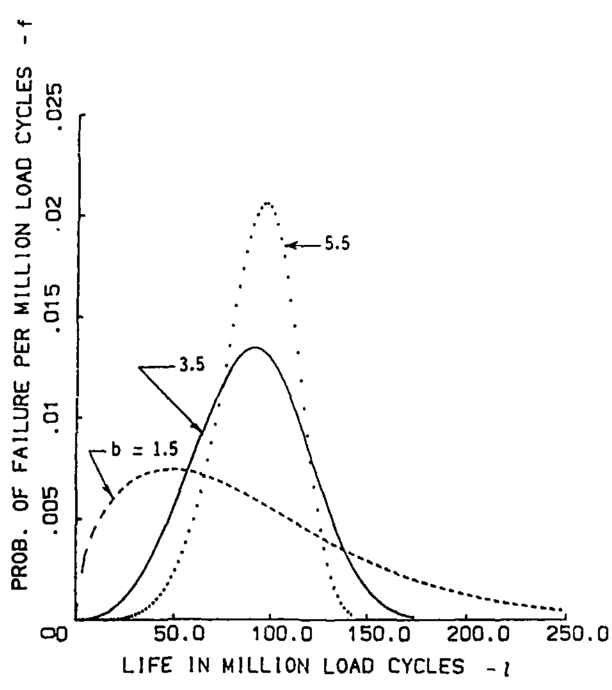

Fig.11 Two-parameter Weibull probability density functions

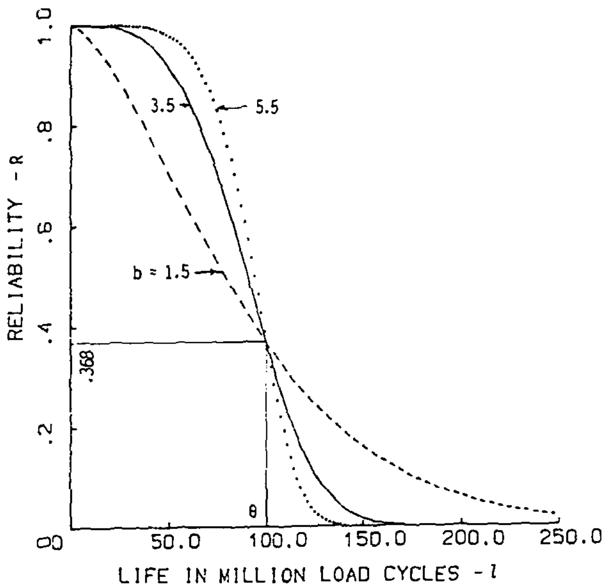

Fig.12 Two-parameter Weibull reliability functions

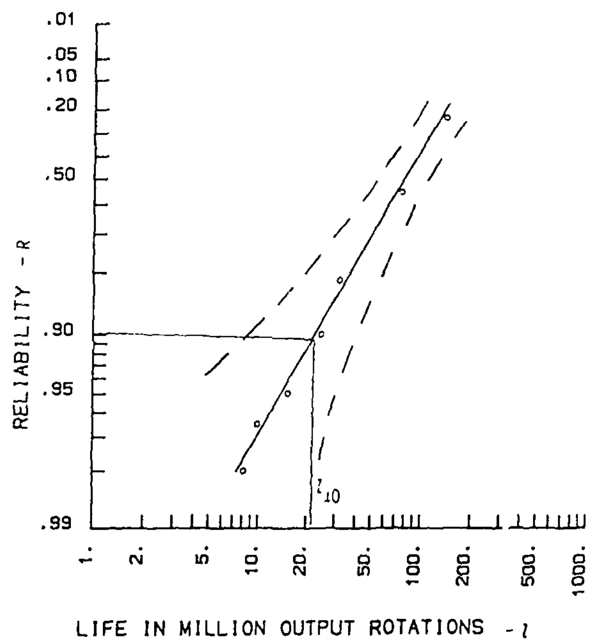

Fig.13 Two-parameter Weibull probability plot

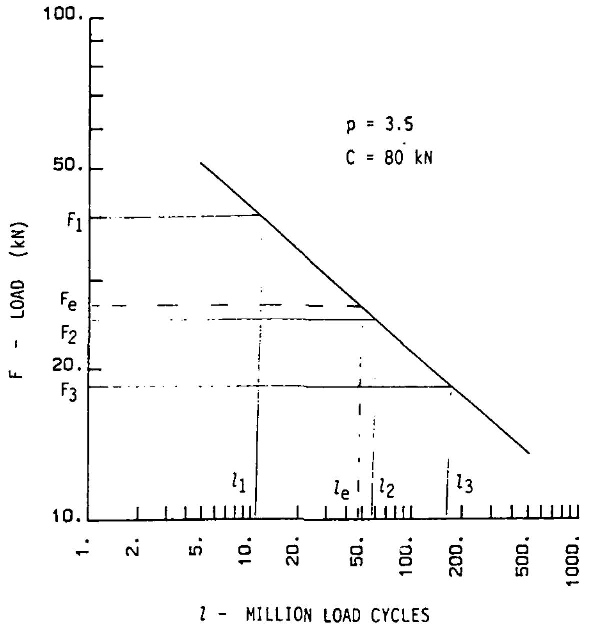

Fig.14 Component load-life curve

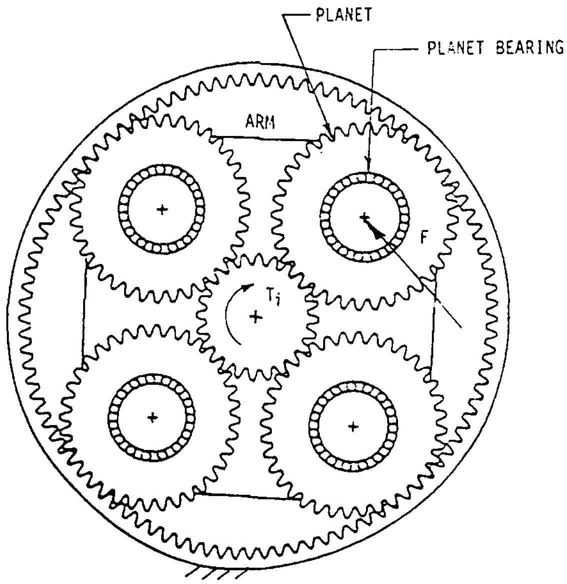

Fig.15 Planet bearing application

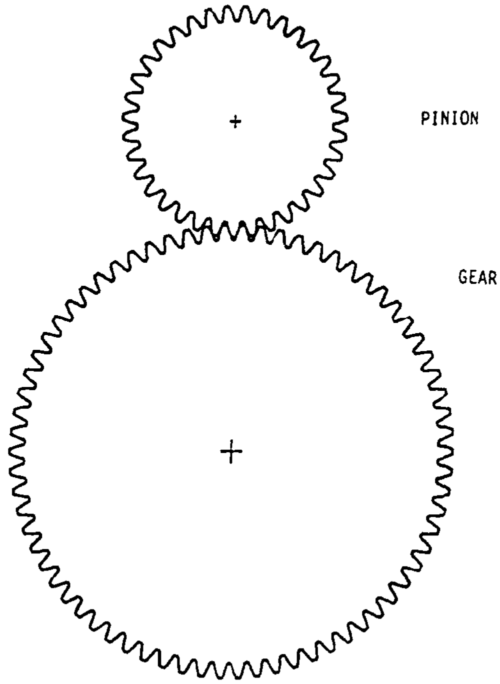

Fig.16 Gear application

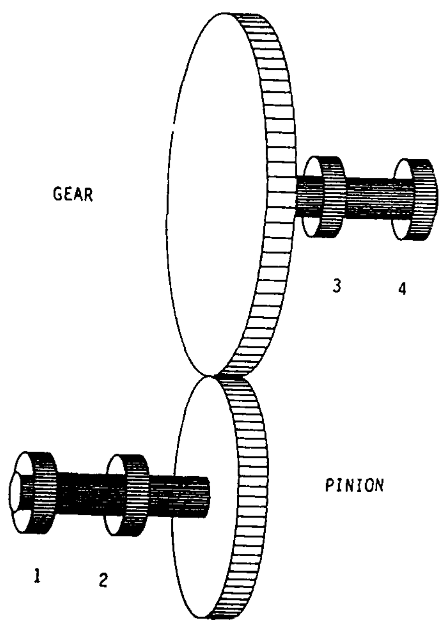

Fig.17 Single mesh drive system

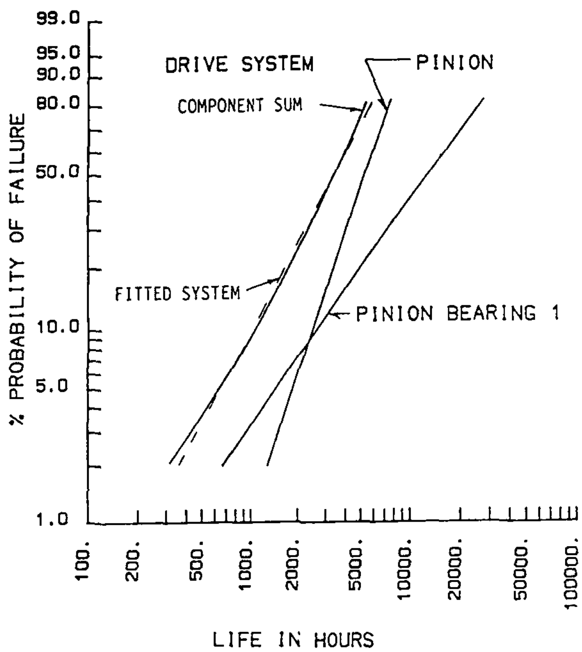

Fig.18 Single mesh drive system reliability

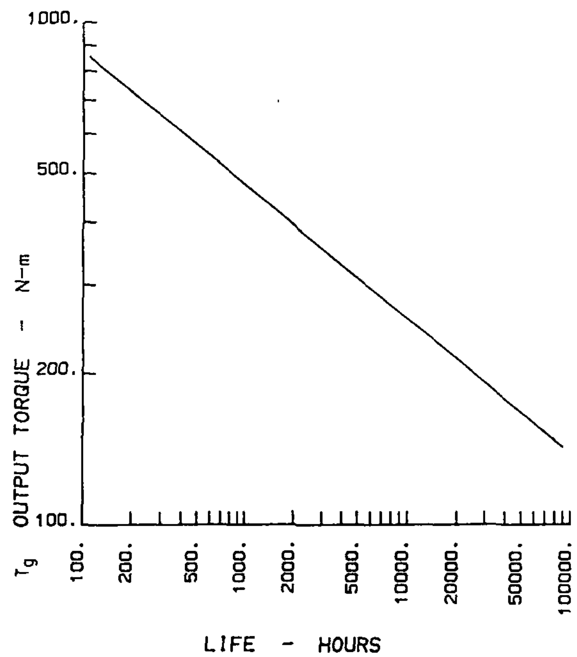

Fig.19 Single mesh drive system load-life curve

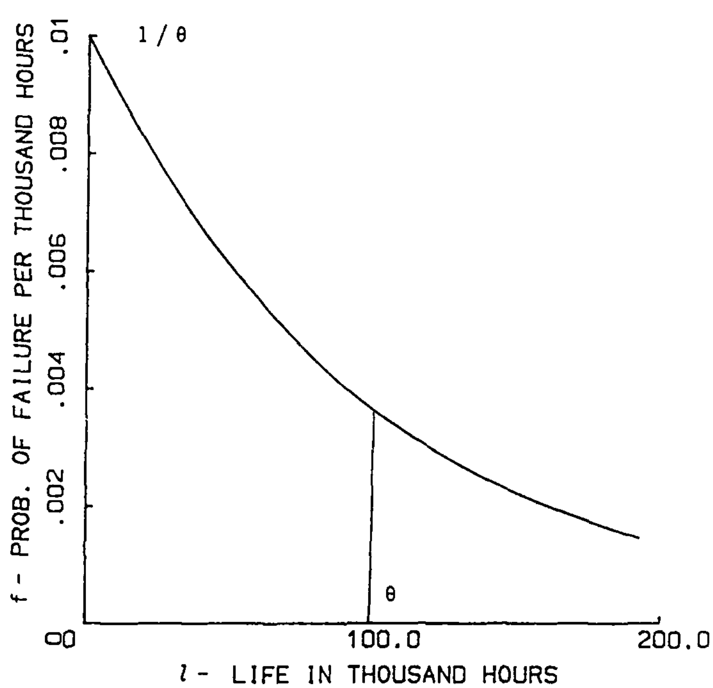

Fig.20 Exponential distribution probability density function with a characteristic life of 100,000 hours

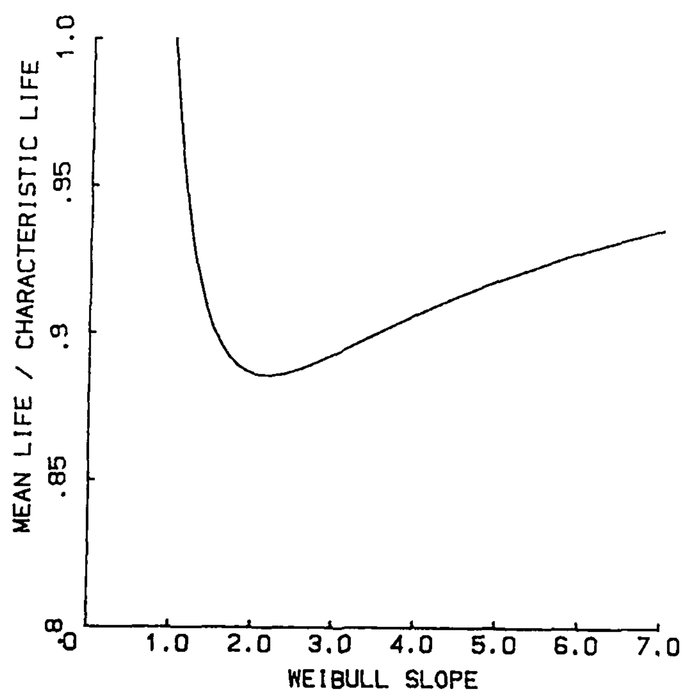

Fig.21 Two-parameter Weibull mean life ratio

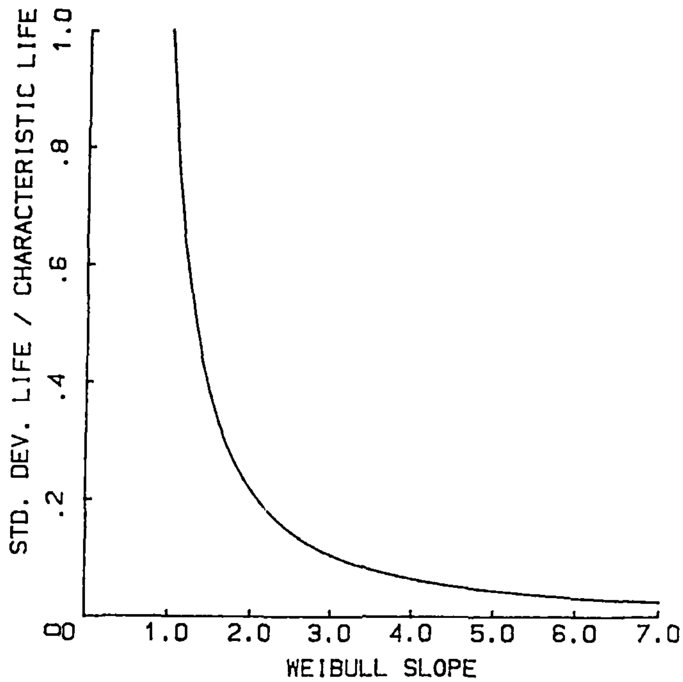

Fig.22 Two-parameter Weibull standard deviation life ratio

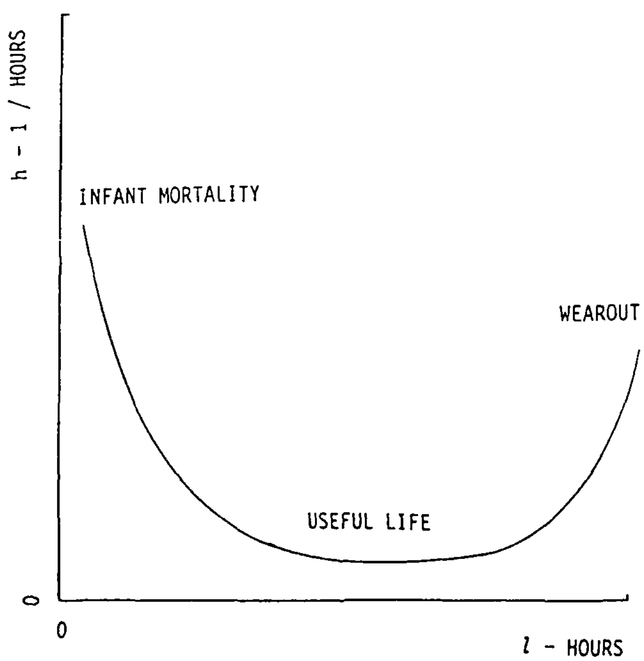

Fig.23 Bath tub mortality curve

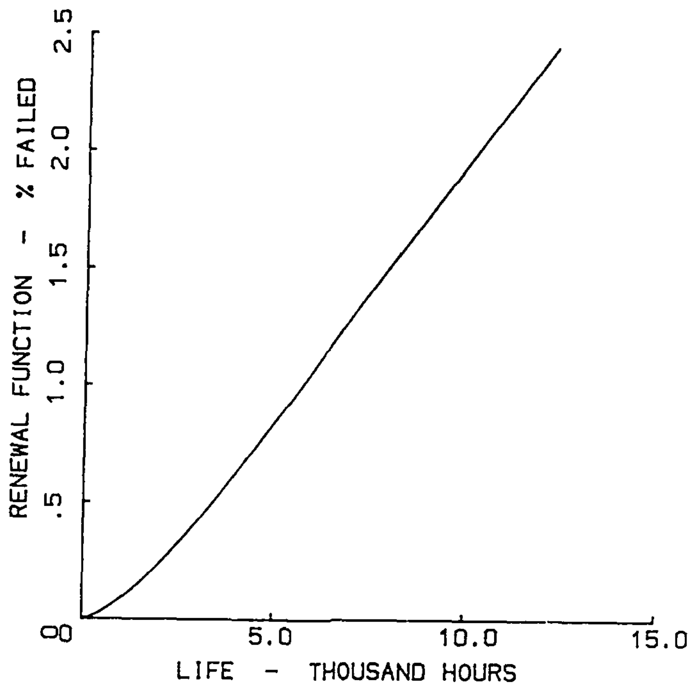

Fig.24 Renewal function mean for a two-parameter Weibull distribution with $\theta = 5{,}000$ hours and $b = 1.5$

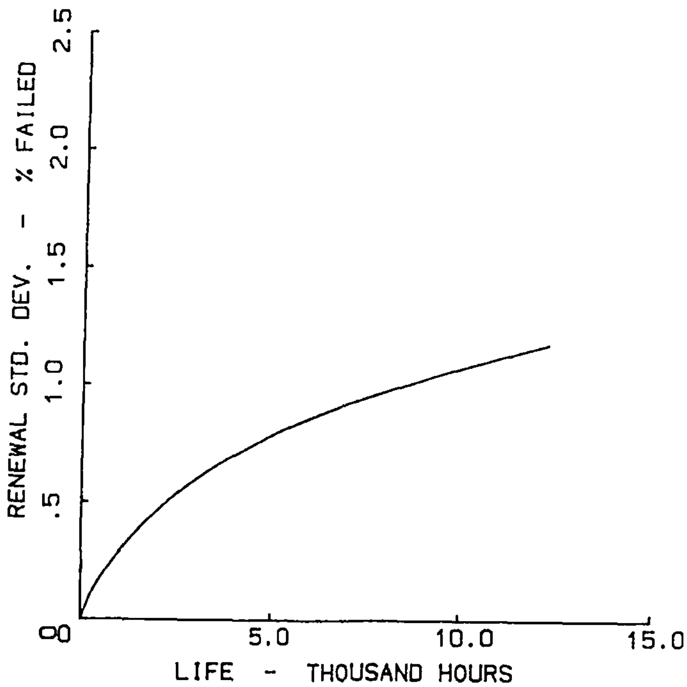

Fig.25 Renewal function standard deviation for a two-parameter Weibull distribution with $\theta = 5{,}000$ hours and $b = 1.5$

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14. Abstract

This report consists of two chapters of an originally intended comprehensive volume on Transmission Systems for Power Transfer in Helicopters and Turboprops which could not be prepared as a whole. After a short introduction, the first chapter deals with lessons learned from accident data of large civil transport helicopters, and with recent improvements in design and component technology, including recommendations and safety benefits achievable by monitoring systems. In the second chapter, statistics of drive system life and reliability were discussed, allowing a proper evaluation of drive system designs and the understanding and control of mean life and life scatter of a drive system at the design stage.

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